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MOTION PLANNING OF UNCERTAIN FULLY-ACTUATED DYNAMICAL SYSTEMS—AN INVERSE DYNAMICS FORMULATION

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ABSTRACT

This work presents a novel nonlinear programming based motion planning framework that treats uncertain fully-actuated dynamical systems described by ordinary differential equations. Uncertainty in multibody dynamical systems comes from various sources, such as: system parameters, initial conditions, sensor and actuator noise, and external forcing. Treatment of uncertainty in design is of paramount practical importance because all real-life systems are affected by it; ignoring uncertainty during design may lead to poor robustness and suboptimal performance. System uncertainties are modeled using Generalized Polynomial Chaos and are solved quantitatively using a least-square collocation method. The computational efficiency of this approach enables the inclusion of uncertainty statistics in the nonlinear programming optimization process. As such, new design questions related to uncertain dynamical systems can now be answered through the new framework.

Specifically, this work presents the new framework through an inverse dynamics formulation where deterministic state trajectories are prescribed and uncertain actuator inputs are quantified. The benefits of the ability to quantify the resulting actuator uncertainty are illustrated in a time optimal motion planning case-study of a serial manipulator pick-and-place application. The resulting design determines a feasible time optimal motion plan—subject to actuator and obstacle avoidance constraints—for all possible systems within the probability space. The forward dynamics formulation (using deterministic actuator inputs and uncertain state trajectories) is presented in a companion paper.

1 INTRODUCTION

Design engineers cannot quantify exactly every aspect of a given system. These uncertainties frequently create difficulties in accomplishing design goals and can lead to poor robustness and suboptimal performance. Tools that facilitate the analysis and characterization of the effects of uncertainties enable designers to develop more robustly performing systems. The need to analyze the effects of uncertainty is particularly acute when designing dynamical systems. Frequently, engineers do not account for various uncertainties in their design in order to save time and to reduce costs. However, this simply delays, or hides, the cost which is inevitably incurred down-stream in the design flow; or worse, after the system has been deployed and fails to meet the design goals. Ultimately, if a robust system design is to be achieved, uncertainties must be accounted for up-front during the design process.

This work presents a novel nonlinear programming (NLP) based motion planning framework that treats uncertain fully-actuated dynamical systems described by ordinary differential equations (ODEs). System uncertainties, such as parameters, initial conditions, sensor/actuator noise, or forcing functions, are modeled using Generalized Polynomial Chaos (gPC) and are solved quantitatively using a least-square collocation method (LSCM). The computational efficiencies gained by gPC and LSCM enable the inclusion of uncertainty statistics in the NLP optimization process.

Specifically, this work presents the new framework through an *inverse dynamics* formulation where deterministic state trajectories are prescribed and uncertain actuator inputs are quantified. The benefits of the ability to quantify the resulting actuator uncertainty are illustrated in a *time optimal* motion planning case-study of a serial manipulator pick-and-place application. The resulting design determines a *time optimal* motion plan—subject to actuator and obstacle avoidance constraints—for all possible systems within the probability space.

The companion formulation based on *uncertain forward dynamics* is presented by the authors in [1]. Application of the *uncertain forward dynamics* has particular advantages for force controlled systems, while the *uncertain inverse dynamics* formulation presented in this work is more suitable for configuration/position controlled systems.

It's important to point out that the new framework is not dependent on the specific formulation of the dynamical equations of motion (EOMs); formulations such as, Newtonian, Lagrangian, Hamiltonian, and Geometric methodologies are all applicable. This work applies the analytical Lagrangian EOM formulation which is briefly introduced in Section 2; Section 3 briefly discusses the well studied deterministic motion planning problem; Section 4 reviews the Generalized Polynomial Chaos methodology for uncertainty quantification when using *inverse dynamics*; Section 5 introduces the new framework for motion planning of uncertain fully-actuated dynamical systems based on an *uncertain inverse dynamics* formulation; finally, Section 6 illustrates the strengths of the new framework through a serial manipulator pick-and-place application which is followed by concluding remarks in Section 7.

2 MULTIBODY INVERSE DYNAMICS

As a very brief overview, the Euler-Lagrange ODE formulation for a multibody dynamical system can be described by [2, 3],

$$\begin{aligned} \boldsymbol{M}\big(\boldsymbol{q}(t),\boldsymbol{\theta}(t)\big)\ddot{\boldsymbol{q}}(t) + \boldsymbol{C}\big(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t),\boldsymbol{\theta}(t)\big)\dot{\boldsymbol{q}}(t) \\ + \boldsymbol{N}\big(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t),\boldsymbol{\theta}(t)\big) \\ = \boldsymbol{\mathcal{F}}\big(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t),\ddot{\boldsymbol{q}}(t),\boldsymbol{\theta}(t)\big) = \boldsymbol{\tau}(t) \end{aligned} \tag{1}$$

where $\boldsymbol{q}(t) \in \mathbb{R}^{n_d}$ are independent generalized coordinates equal in number to the number of degrees of freedom, n_d (the illustrating case study uses relative joint angles but the formulation is not limited to such a choice); $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^{n_d}$ the rates of the generalized coordinates and $\ddot{\boldsymbol{q}}(t)$ are the associated accelerations—using Newton's *dot* notation for a time derivative; $\boldsymbol{\theta}(t) \in \mathbb{R}^{n_p}$ includes system parameters of interest (specifically, those with uncertainty as described in Section 4); $M\big(\boldsymbol{q}(t),\boldsymbol{\theta}(t)\big) \in \mathbb{R}^{n_d \times n_d}$ is the square positive definite inertia matrix; $\boldsymbol{C}\big(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t),\boldsymbol{\theta}(t)\big) \in \mathbb{R}^{n_d \times n_d}$ includes centrifugal, gyroscopic and Coriolis effects; $\boldsymbol{N}\big(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t),\boldsymbol{\theta}(t)\big) \in \mathbb{R}^{n_d}$ the generalized gravitational and joint forces; and $\boldsymbol{\tau}(t) \in \mathbb{R}^{n_i}$ are the n_i applied input wrenches. (For notational brevity, all future equations will drop the explicit time dependence.)

The trajectory of the system is determined by solving (1) as an initial value problem, where $\boldsymbol{q}(0) = \boldsymbol{q}_0$ and $\dot{\boldsymbol{q}}(0) = \dot{\boldsymbol{q}}_0$. Also, the system measured outputs are defined by,

$$\boldsymbol{y} = \boldsymbol{\mathcal{O}}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\theta}) \tag{2}$$

where $\boldsymbol{y} \in \mathbb{R}^{n_o}$ with n_o equal to the number of outputs.

A close inspection of (1) shows that the wrench inputs of fully-actuated systems can be calculated through direct algebraic evaluations if the state trajectories are "known", or prescribed,

$$\boldsymbol{\tau} = \boldsymbol{\mathcal{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\theta}) \tag{3}$$

This is the *inverse dynamics* (ID) formulation and does not require numerical integration and can result in significant computational savings.

3 DETERMINISTIC INVERSE DYNAMICS MOTION PLANNING

The task of dynamic system motion planning is a well studied topic; it aims to determine a state, or input, trajectory to realize some prescribed objective. Sampled-based motion planning formulations, such as Rapid-exploring Random Trees (RRTs), primarily focus on finding a feasible solution [4-6]; where nonlinear programming formulations seek to determine at least a local optimal solution [7-12].

Use of an *inverse dynamics* formulation requires a practitioner to define the state trajectory, $\{\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}\}$, over the entire motion plan. This is an infinite dimensional problem. Parameterized trajectories are commonly used to reduce the problem to a finite dimensional search. For example, the system's configuration, $\boldsymbol{q}$, can be represented with B-Splines,

$$q(\boldsymbol{P},t) = \sum_{i=0}^{n_{sp}} \beta^{i,\wp-1}(t)\boldsymbol{p}^i \tag{4}$$

where $\boldsymbol{p} \in \mathbb{R}^{n_d}$ are n_{sp} control points; β and $\wp$ are the B-Splines' basis functions and degree, respectively. The corresponding parameterizations for $\{\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}\}$ are also B-Splines derived from $\boldsymbol{q}(\boldsymbol{P},t)$ [13] ; $\boldsymbol{P} \in \mathbb{R}^{n_{sp} \times n_d}$ is an vector of control points, $\boldsymbol{p}$.

Once the state trajectories have been parameterized the NLP-based deterministic motion planning problem may be formulated as,

$$\begin{aligned} \min_{x=\{\boldsymbol{P}\}} \quad & J \\ \text{s.t.} \quad & \boldsymbol{\tau} = \boldsymbol{\mathcal{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\theta}) \\ & \mathbf{y} = \boldsymbol{\mathcal{O}}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta}) \\ & \boldsymbol{\mathcal{C}}(\boldsymbol{y},\boldsymbol{\tau},\boldsymbol{\theta}) \leq \mathbf{0} \end{aligned} \tag{5}$$

where the state's explicit dependence on their associated control points has been dropped for notational brevity. Equation (5) seeks to find the control points $\boldsymbol{P}$ that minimize some prescribed objective function, J, while being subject to the *inverse dynamic* constraints defined in (3). Additional constraints may also be defined; for example, maximum/minimum actuator and system parameter limits or physical system geometric limits can be represented as inequality relations, $\boldsymbol{\mathcal{C}}(\boldsymbol{y},\boldsymbol{\tau},\boldsymbol{\theta}) \leq \mathbf{0}$.

The literature contains a variety of objective function definitions for J when used in a motion planning setting. Some commonly defined objective functions are,

$$J_{D1} = t_f \tag{6}$$

$$\mathrm{J_{D2}} = \sum_{i=1}^{n_i} \tau_i^{\,2}, \quad \forall t \tag{7}$$

$$\mathrm{J_{D3}} = \sum_{i=1}^{n_i} |\tau_i \dot{q}_{\mathrm{i}}|, \quad \forall t \tag{8}$$

$$\mathrm{J_{D4}} = \sum_{i=1}^{n_i} \dot{\tau}_i^{\,2}, \quad \forall t \tag{9}$$

where (6) represents a *time optimal* design; (7) minimizes effort, (8) power, and (9) jerk.

The solution to (5) produces an optimal motion plan under the assumption that all system properties are known (i.e., (3) is completely deterministic). The primary contribution of this work is the presentation of a variant of (5) that allows (3) to contain uncertainties of diverse types (e.g., parameters, initial conditions, sensor/actuator noise, or forcing functions). The following section will briefly introduce Generalized Polynomial Chaos (gPC) which is used to model the uncertainties and to quantify the resulting uncertain input wrenches.

4 GENERALIZED POLYNOMIAL CHAOS

Generalized Polynomial Chaos (gPC), first introduced by Wiener [14], is an efficient method for analyzing the effects of uncertainties in second order random processes [15]. This is accomplished by approximating a source of uncertainty, θ, with an infinite series of weighted orthogonal polynomial bases called Polynomial Chaoses. Clearly an infinite series is impractical; therefore, a truncated set of $p_o + 1$ terms is used with $p_o \in \mathbb{N}$ representing the *order* of the approximation. Or,

$$\theta(\xi) = \sum_{j=0}^{p_o} \theta^j \psi^j\left(\xi(\omega)\right) \tag{10}$$

where $\theta^j \in \mathbb{R}$ represent known stochastic coefficients; $\psi^j \in \mathbb{R}$ represent individual single dimensional orthogonal basis terms (or modes); $\xi(\omega) \in \mathbb{R}$ is the associated random variable for θ that maps the random event $\omega \in \Omega$, from the sample space, Ω, to the domain of the orthogonal polynomial basis (e.g., $\xi: \Omega \to [-1,1]$).

Polynomial chaoses are orthogonal with respect to the ensemble inner product,

$$\langle \psi^i(\xi), \psi^j(\xi) \rangle = \int_{-1}^{1} \psi^i(\xi) \psi^j(\xi) w(\xi) d\xi = 0, \quad \text{for } i \neq j \tag{11}$$

where $w(\xi)$ is the weighting function that is equal to the joint probability density function of the random variable ξ. Also, $\langle \Psi^j, \Psi^j \rangle = 1, \forall j$ when using *normalized basis*; *standardized basis* are constant and may be computed off-line for efficiency using (11).

Generalized Polynomial Chaos can be applied to multibody dynamical systems described by differential equations [16, 17]; where sources of uncertainty, such as parameters, initial conditions, sensor/actuator noise, or forcing functions, are all treated in a unified manner. The presence of uncertainty in the system results in either uncertain states, as in a *forward dynamics* formulation (1) [1], or uncertain inputs, as in an *inverse dynamics* formulation (3). Therefore, proceeding with the *inverse dynamics* formulation, the uncertain input wrenches can be approximated in a similar fashion as (10),

$$\tau_i(\xi; t) = \sum_{j=0}^{n_b} \tau_i^j(t) \Psi^j(\xi), \qquad i = 1 \dots n_i \tag{12}$$

where $\tau_i^j(t) \in \mathbb{R}^{n_b}$ again represents the stochastic coefficients—for the i^{th} input wrench—but are now unknown functions of time, with $n_b \in \mathbb{N}$ representing the number of basis terms in the approximation.

The stochastic basis of the inputs may be multidimensional in the event there are multiple sources of uncertainty. The multidimensional basis functions are represented by $\Psi^j \in \mathbb{R}^{n_b}$. Additionally, $\boldsymbol{\xi}$ becomes a vector of random variables, $\boldsymbol{\xi} = \{\xi_1, \dots, \xi_{n_p}\} \in \mathbb{R}^{n_p}$ and maps the sample space, Ω, to an n_p dimensional cuboid, $\boldsymbol{\xi}: \Omega \to [-1,1]^{n_p}$ (as in the example of Jacobi chaoses).

The multidimensional basis is constructed from a product of the single dimensional basis in the following manner,

$$\Psi^j = \psi_1^{i_1} \psi_2^{i_2} \dots \psi_{n_p}^{i_{n_p}}, \quad i_k = 0 \dots p_o, k = 1 \dots n_p \tag{13}$$

where subscripts represent the uncertainty source and superscripts represent the associated basis term (or mode). A complete set of basis may be determined from a full tensor product of the single dimensional bases. This results in an excessive set of $(p_o + 1)^{n_p}$ basis terms. Fortunately, the multidimensional sample space can be spanned with a minimal set of $n_b = \frac{(n_p + p_o)!}{n_p!\, p_o!}$ basis terms. The minimal basis set can be determined by the products resulting from these index ranges,

$$i_1 = 0 \dots p_o,$$
$$i_2 = 0 \dots (p_o - i_1), \dots,$$
$$i_{n_p} = 0 \dots \left(p_o - i_1 - i_2 - \dots - i_{(n_p - 1)}\right)$$

The number of multidimensional terms, n_b, grows quickly with the number of uncertain parameters, n_p, and polynomial order, p_o. Sandu et. al. showed that gPC is most appropriate for modeling systems with a relatively low number of uncertainties [16, 17] but can handle large nonlinear uncertainty magnitudes.

Substituting (10) and (12) into (3) produces the following *uncertain inverse dynamics* (UID),

$$\sum_{j=0}^{n_b} \tau_i^j(t) \Psi^j(\boldsymbol{\xi}) = \boldsymbol{\mathcal{F}}\left(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \sum_{j=0}^{p_o} \theta_k^j(t)\, \psi_k^j(\xi_k) \right), \quad i = 1 \dots n_i, k = 1 \dots n_p \tag{14}$$

where the unknowns are now the $n_b n_i$ stochastic input coefficients, $\tau_i^j(t)$.

It is instructive to notice how time and randomness are decoupled within a single term after the gPC expansion. Only the stochastic coefficients are dependent on time, and only the basis terms are dependent on the n_b random variables, $\boldsymbol{\xi}$.

The Galerkin Projection Method (GPM) is a commonly used method for solving (14), however, this is a very intrusive technique and requires a custom formulation of the *inverse dynamic* EOMs. As an alternative, sample-based collocation techniques can be used without the need to modify the base EOMs.

Sandu et. Al. [16, 18] showed that the collocation method solves (14) by solving (3) at a set of points, ${}_k\boldsymbol{\mu} \in \mathbb{R}^{n_p}$, $k = 1 \dots n_{cp}$, selected from the n_p dimensional domain of the random variables $\boldsymbol{\xi} \in \mathbb{R}^{n_p}$. Meaning, at any given instance in time, the random variables' domain is sampled and solved n_{cp} times with $\boldsymbol{\xi} = {}_k\boldsymbol{\mu}$ (updating the approximations of all sources of uncertainty for each solve), then the unknown stochastic input coefficients τ_i^j can be determined at that given time instance. This can be accomplished by defining the intermediate variables,

$${}_kT_i\left(t;\, {}_k\boldsymbol{\mu}\right) = \sum_{j=0}^{n_b} \tau_i^j(t) \Psi^j\left({}_k\boldsymbol{\mu}\right), \quad i = 1 \dots n_i, k = 0 \dots n_{cp} \tag{15}$$

and substitute them into (14). This yields,

$${}_kT_i\left(t;\, {}_k\boldsymbol{\mu}\right) = \mathcal{F}\left(q, \dot{q}, \ddot{q},\, {}_k\Theta_r\left(t;\, {}_k\boldsymbol{\mu}\right)\right), \quad i = 1 \dots n_i, k = 0 \dots n_{cp}, r = 1 \dots n_p \tag{16}$$

where,

$${}_k\Theta_r\left(t;\, {}_k\boldsymbol{\mu}\right) = \sum_{j=0}^{p_o} \theta_r^j(t)\, \psi^j\left({}_k\boldsymbol{\mu}_i\right), \quad k = 0 \dots n_{cp}, r = 1 \dots n_p \tag{17}$$

Equation (16) provides a set of n_{cp} independent equations whose solutions determine the stochastic coefficients, $\tau_i^j(t)$. This is accomplished by recalling the relationship of the stochastic coefficients to the solutions, ${}_kT_i$, shown in (15). In matrix notation (15) can be expressed for all inputs,

$$\boldsymbol{T}_i = \left(\tau_i(t)\right)^T \boldsymbol{\Psi}(\boldsymbol{\mu}), \qquad i = 1 \dots n_i \tag{18}$$

where the matrix,

$$A_{k,j} = \boldsymbol{\Psi}^j\left({}_k\boldsymbol{\mu}\right), \qquad j = 0 \dots n_b, k = 0 \dots n_{cp} \tag{19}$$

is defined as the *collocation matrix*. It's important to note that $n_b \leq n_{cp}$. The stochastic coefficients can now be solved for using (18),

$$\tau_i^j(t) = \boldsymbol{A}^{\#}\boldsymbol{T}_i, \qquad i = 1 \dots n_i, j = 0 \dots n_b \tag{20}$$

where $A^{\#}$ is the pseudo inverse of A if $n_b < n_{cp}$. If $n_b = n_{cp}$, then (20) is simply a linear solve. However, [18] presented the least-squares collocation method (LSCM) where the stochastic state coefficients are solved for, in a least squares sense, using (20) when $n_b < n_{cp}$. [18] also showed that as $n_{cp} \to \infty$ the LSCM approaches the GPM solution; where by selecting $3n_b \leq n_{cp} \leq 4n_b$ the greatest convergence benefit is achieved with minimal computational cost. LSCM also enjoys the same exponential convergence rate as $p_o \to \infty$.

The unintrusive nature of the LSCM sampling approach is arguably its greatest benefit; (3) may be repeatedly solved without modification. Also, there are a number of methods for selecting the collocation points and the interested reader is recommended to consult [16, 18-21] for more information.

5 UNCERTAIN INVERSE DYNAMICS MOTION PLANNING

Little work is found in the literature addressing motion planning for uncertain systems. The literature thus far has primarily addressed sensor and/or actuator noise [4, 22] and frequently only treats the system's kinematics [23, 24].

In [25], Kewlani presents an RRT planner for mobility of robotic systems based on gPC but refers to it as a stochastic response surface method (SRSM). Kewlani's work is similar in spirit to this work, however, the main difference is Kewlani's solution is developed only for determining a feasible motion plan. The motion planning framework of this paper is formulated within a NLP setting and thus benefits from the more efficient gradient-based searching techniques providing at least a locally optimal design.

As such, the new NLP-based framework for motion planning of uncertain fully-actuated multibody dynamical systems, formulated with *uncertain inverse dynamics*, is,

$$\begin{aligned} &\min_{\boldsymbol{x}=\{\boldsymbol{P}\}} && J(t;\boldsymbol{\xi}) \\ &\text{s.t.} && \boldsymbol{\tau}(t;\boldsymbol{\xi}) = \boldsymbol{\mathcal{F}}\left(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\theta}(t;\boldsymbol{\xi})\right) \\ &&& \mathbf{y} = \boldsymbol{\mathcal{O}}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\theta}) \\ &&& \boldsymbol{\mathcal{C}}\left(\boldsymbol{y},\boldsymbol{\tau},\boldsymbol{\theta}(t;\boldsymbol{\xi})\right) \leq \mathbf{0} \end{aligned} \tag{21}$$

where (21) is a reformulation of (5) with uncertain actuator inputs. The most interesting part of (21) comes in the definition of the objective function terms and constraints. These terms now have the ability to approach the design accounting for uncertainties by way of expected values, variances, and standard deviations. Recalling the definitions of an expected value and variance, (7)–(9) may be redefined statistically:

$$\begin{aligned} \mathrm{J}_{S1} &= E\left[\sum_{i=1}^{n_i} \mathrm{z}_i\left(\tau_i(\boldsymbol{\xi})\right)^2\right] \\ &= \textstyle\sum_{i=1}^{n_i}\sum_{j=0}^{n_b} \mathrm{z}_i\left(\tau_i^j\right)^2 \langle \Psi^j, \Psi^j \rangle, \ \forall t \end{aligned} \tag{22}$$

$$\begin{aligned} \mathrm{J}_{S2} &= E\left[\sum_{i=1}^{n_i} \left|\mathrm{z}_i\tau_i(\boldsymbol{\xi})\mathrm{y}_i(\boldsymbol{\xi})\right|\right] \\ &= \textstyle\sum_{i=1}^{n_i}\sum_{j=0}^{n_b} \left|\mathrm{z}_i\tau_i^j\mathrm{y}_i^j\langle \Psi^j, \Psi^j \rangle\right|, \ \forall t \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{J}_{S3} &= E\left[\sum_{i=1}^{n_i} \mathrm{z}_i\left(\dot{\tau}_i(\boldsymbol{\xi})\right)^2\right] \\ &= \textstyle\sum_{i=1}^{n_i}\sum_{j=0}^{n_b} \mathrm{z}_i\left(\dot{\tau}_i^j\right)^2 \langle \Psi^j, \Psi^j \rangle, \ \forall t \end{aligned} \tag{24}$$

where $\mathbf{z}$ is a vector of optional scalarization weights; (22) encapsulates expected effort; (23) expected power; and (24) expected jerk. Notice that due to the orthogonality of the polynomial basis these computations result in a reduced set of efficient operations on the respective stochastic coefficients.

The inequality constraints may also benefit from added statistical information. When using *inverse dynamics*, the input wrenches are uncertain. This uncertain quantity may also be bound by physical limits of the actuator; as an example, inequality constraints may be formulated as,

$$\boldsymbol{\mathcal{C}} = \begin{cases} \mu_{\tau_i} + \sigma_{\tau_i} \leq \bar{\tau} \\ \underline{\tau} \leq \mu_{\tau_i} - \sigma_{\tau_i} \end{cases}, \qquad i = 1,2 \tag{25}$$

where the mean $\mu_{\tau_i} = \tau_i^0$ is defined as in (22)–(24), the standard deviation $\sigma_{\tau_i} = \sqrt{\sum_{j=1}^{n_b} \tau_i^j}$ is the root of the variance, and $\{\underline{\tau}, \bar{\tau}\}$ are the minimum/maximum input bounds respectively.

Deterministic terms, such as (6)–(9), may be combined with appropriately selected statistically based terms, such as (22)–(25), to form a final motion planning problem. This will be illustrated in the serial manipulator case-study in the following section.

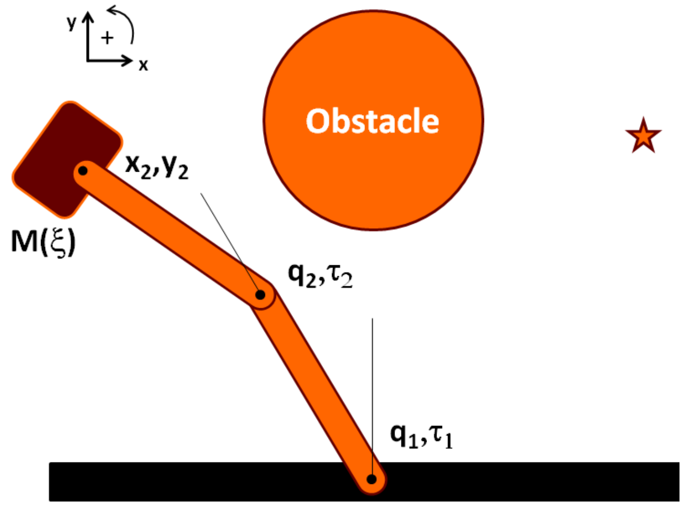

Figure 1—A simple illustration of the fully-actuated uncertain inverse dynamics motion planning formulation; this problem aims to determine a time optimal motion plan subject to input wrench and geometric collision constraints. This is an uncertain system due to the uncertain mass of the payload.

6 A SERIAL MANIPULATOR PICK-AND-PLACE CASE-STUDY

As an illustration of (21), the serial manipulator "pick-and-place" problem will be used (see Figure 1). The design objective is to minimize the time it takes to move the manipulator from its initial configuration, $\boldsymbol{q}_0$, to the target configuration, $\boldsymbol{q}_{t_f}$. This results in a deterministic objective function, $J = t_f$, which is frequently referred to as a *time optimal* design. However, the payload mass, $M(\xi)$, is defined to be uncertain rendering the system dynamics uncertain. Since the uncertain serial manipulator is a fully actuated system, where the joints $\boldsymbol{q} = \{q_1, q_2\}$ are actuated with the input wrenches $\boldsymbol{\tau} = \{\tau_1, \tau_2\}$, the motion planning problem may be appropriately defined by (21).

By parameterizing the deterministic joint trajectories with B-Splines, as in (4), (21) results in a finite search problem seeking for spline control points, **P**, that minimize the trajectory time, t_f. Therefore, the problem's optimization variables are $\boldsymbol{x} = \{\mathbf{P}, t_f\}$.

The actuators are bounded in their torque supply and the manipulator should neither hit the wall it's mounted to nor the obstacle. The constraints may therefore be defined as,

$$\boldsymbol{C}: \begin{cases} \mu_{\tau_i} + \sigma_{\tau_i} \le \bar{\tau} \\ \underline{\tau} \le \mu_{\tau_i} - \sigma_{\tau_i} \\ -y_1 \le 0 \\ -y_2 \le 0 \\ -\mathfrak{D}_{i,j} \le 0 \end{cases} \tag{26}$$

where $i = 1{,}2$ and $j = obstacle$ for the signed distance, $\mathfrak{D}_{i,j}$, measured from each link of the serial manipulator to the obstacle.

Notice the bounding constraints on the input wrenches are defined by their statistical mean and standard deviations, as in (25), to quantify their uncertainty. Ideally these constraints would be defined by the extremes of the wrench distribution (i.e., the *supremum* and *infimum*), however, due to their computational complexity the approximation by the mean and standard deviation, as in (26), is used.

Since the state trajectories are deterministic, the signed obstacle avoidance constraints, $-\mathfrak{D}_{i,j} \le 0$, and Cartesian wall avoiding constraints, $-y_1, -y_2 \le 0$, are deterministically defined.

This formulation allows a design engineer to answer the question,

> *Given actuator and obstacle constraints, what is the "time optimal" motion plan that accounts for all possible systems within the probability space?*

Without accounting for the uncertainty directly in the dynamics and motion planning formulations, design engineers would have a difficult time answering this question.

The solution to this problem with the deterministic formulation, as defined in (5), results in a *time optimal* solution of $t_f = 1.12$ seconds; where all system parameters are set equal to one, $\theta_i = 1$ (with SI units); with initial conditions $\boldsymbol{q}(0) = \{\frac{\pi}{6}, \frac{\pi}{6}\}$ and $\dot{\boldsymbol{q}}(0) = \{0, 0\}$ radians; terminal conditions $\boldsymbol{q}(t_f) = \{-\frac{\pi}{6}, -\frac{\pi}{6}\}$ and $\dot{\boldsymbol{q}}(t_f) = \{0, 0\}$ radians; and $\underline{\tau} = -10, \bar{\tau} = 10$ (Nm). The resulting optimal input wrench time history is shown in Figure 2.

The solution from the new formulation, as defined in (21) with constraints defined by (26), results in a *time optimal* solution of $t_f = 1.2$ seconds; where all system parameters and initial/terminal conditions are defined the same as in the deterministic problem. The only difference in this problem definition, as compared to the deterministic problem, is the uncertain payload mass modeled with a uniform distribution having a 1 (kg) mean and 0.5 (kg) variance. The resulting optimal uncertain input wrench time history is illustrated in Figure 3; where each input wrench is displaying its mean value and bounding $\mu_{\tau_i} + \sigma_{\tau_i}$ time histories. Also, the resulting configuration time history for the optimal uncertain motion plan is shown in Figure 4.

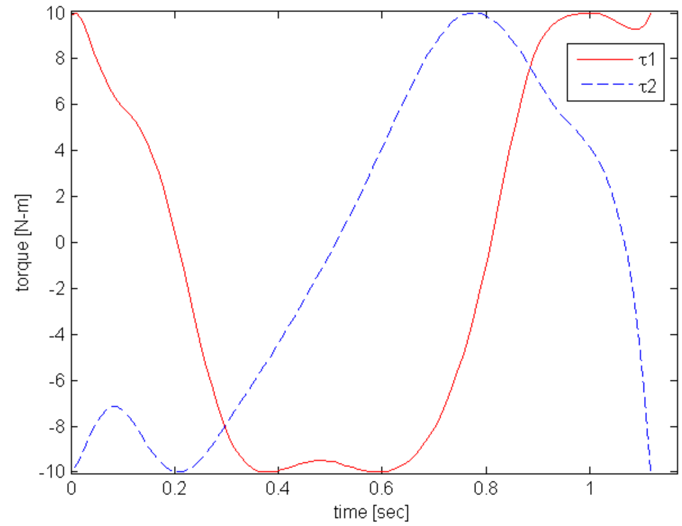

Figure 2—The time optimal input wrench time histories for the deterministic serial manipulator 'pick-and-place' problem. This optimal solution resulted in a $t_f = 1.12$ (s).

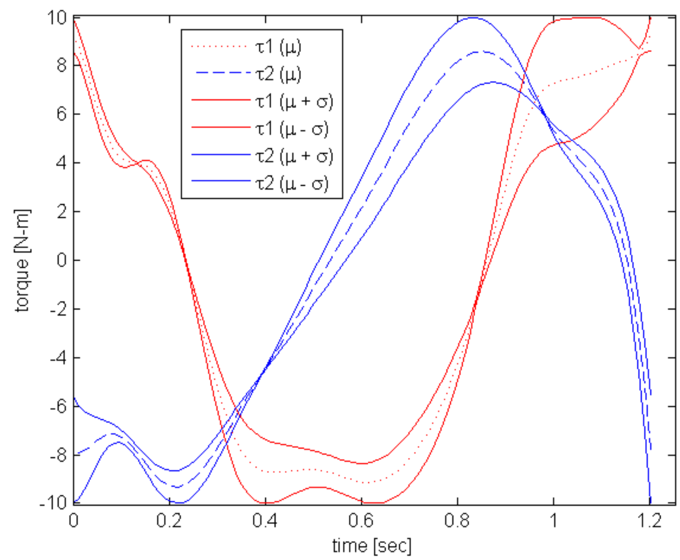

Figure 3—The time optimal uncertain input wrench time histories for the uncertain serial manipulator 'pick-and-place' problem. Each input wrench is displaying its mean value and bounding $\mu_{\tau_i} + \sigma_{\tau_i}$ time histories. This optimal solution resulted in a $t_f = 1.2$ (s).

Therefore, the *time optimal* solution from the uncertain problem resulted in a more conservative answer (1.2 seconds as compared to 1.12 seconds). This is a sensible solution; close inspection of Figure 2 shows the deterministic solution drove the input wrenches to their extreme bounds of +/-10 (Nm) at certain points during the motion profile. Clearly, introducing the uncertain mass to the system affected the amount of input torque required for the system to reliably follow the specified state trajectory. In fact, Figure 3 shows the distribution of input wrenches induced by the uncertain mass. The uncertain optimal motion plan from (21) effectively pushed the input wrench distribution inside the actuation limits, $\{\underline{\tau}, \bar{\tau}\}$; this results in a slower *time optimal* solution, however, all realizable systems within the probability space of the uncertain mass are now guaranteed to satisfy the constraints. In other words, the *time optimal* solution to (21) produces the minimum time for the entire family of systems. Relying only on the contemporary deterministic problem formulation in (5) results in an unrealizable trajectory for a subset of the realizable systems.

Additionally, the author's companion paper [1] presents data showing that use of a parallelized LCSM based gPC in the new framework allows for efficient optimal motion planning of uncertain dynamical systems; where the additional cost reduces as the number of available parallel processors increases.

A final observation is that the *uncertain inverse dynamics* motion planning framework embodied in (21) is most applicable to configuration/position controlled systems, where states are prescribed as they are in (21). However, force controlled systems may be better designed through application of the companion framework based on *uncertain forward dynamics* presented by the authors in [1].

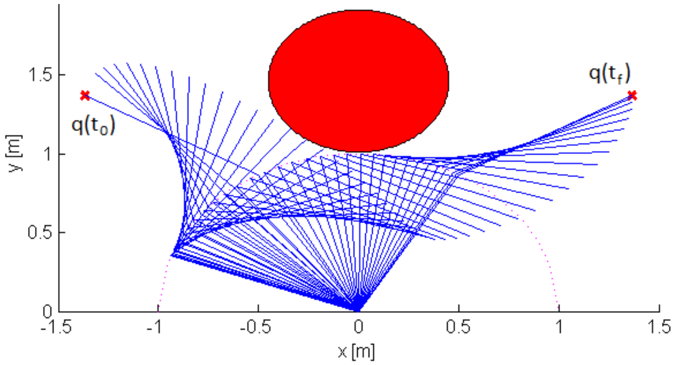

Figure 4—The final optimal configuration time history of the uncertain serial manipulator 'pick-and-place' application involving collision avoidance and actuator constraints.

7 CONCLUSIONS

This work presents a new nonlinear programming based motion planning framework that treats uncertain fully-actuated dynamical systems. The framework allows practitioners to model sources of uncertainty using the Generalized Polynomial Chaos methodology and to solve the *uncertain inverse dynamics* using a least-squares collocation method. The uncertainty aware design is obtained by including statistical information of the *uncertain inverse dynamics* in the NLP's objective function and constraints. The serial manipulator case study illustrated how the new framework produces an optimal design that accounts for the entire family of systems enabling a practitioner to design an optimally performing system that is also robust.

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REFERENCES

[1] Hays, J., Sandu, A., and Sandu, C., 2011, "Motion Planning of Uncertain Fully-Actuated Dynamical Systems—a Forward Dynamics Formulation," ASME IDETC/CIE Conference, Washington, DC, USA, pp. (submitted).

[2] Greenwood, D., 2003, *Advanced Dynamics*, Cambridge Univ Pr,

[3] Murray, R., Li, Z., Sastry, S., and Sastry, S., 1994, *A Mathematical Introduction to Robotic Manipulation*, CRC Press, Inc, Boca Raton, FL, USA.

[4] Lavalle, S., 2006, *Planning Algorithms*, Cambridge Univ Press, New York, NY, USA.

[5] Choset, H., 2005, *Principles of Robot Motion: Theory, Algorithms, and Implementation*, The MIT Press, Cambridge, MA, USA.

[6] Karaman, S., and Frazzoli, E., 2010 (submitted), "Incremental Sampling-Based Algorithms for Optimal Motion Planning," International Journal of Robotics Research, pp.

[7] Park, J., 2007, Industrial Robotics, Programming, Simulation and Applications, Verlag, Croatia, Optimal Motion Planning for Manipulator Arms Using Nonlinear Programming.

[8] Sohl, G. A., and Bobrow, J. E., 2001, "A Recursive Multibody Dynamics and Sensitivity Algorithm for Branched Kinematic Chains," Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control, 123 (Copyright 2002, IEE), pp. 391-9.

[9] Bobrow, J., Martin, B., Sohl, G., Wang, E., Park, F., and Kim, J., 2001, "Optimal Robot Motions for Physical Criteria," Journal of Robotic systems, 18 (12), pp. 785-795.

[10] Sohl, G., 2000, "Optimal Dynamic Motion Planning for Underactuated Robots," PhD thesis, University of California, Irvine.

[11] Xiang, Y., Arora, J., and Abdel-Malek, K., 2010, "Physics-Based Modeling and Simulation of Human Walking: A Review of Optimization-Based and Other Approaches," Structural and Multidisciplinary Optimization, 42 (1), pp. 1-23.

[12] Lee, S. H., Kim, J., Park, F. C., Kim, M., and Bobrow, J. E., 2005, "Newton-Type Algorithms for Dynamics-Based Robot Movement Optimization," Robotics, IEEE Transactions on, 21 (4), pp. 657-667.

[13] Piegl, L. A., and Tiller, W., 1997, *The Nurbs Book*, Springer Verlag, Berlin, Germany.

[14] Wiener, N., 1938, "The Homogeneous Chaos," American Journal of Mathematics, 60 (4), pp. 897-936.

[15] Xiu, D., and Karniadakis, G., 2003, "The Wiener-Askey Polynomial Chaos for Stochastic Differential Equations," pp.

[16] Sandu, A., Sandu, C., and Ahmadian, M., 2006, "Modeling Multibody Systems with Uncertainties. Part I: Theoretical and Computational Aspects," Multibody System Dynamics, 15 (4), pp. 369-391.

[17] Sandu, C., Sandu, A., and Ahmadian, M., 2006, "Modeling Multibody Systems with Uncertainties. Part Ii: Numerical Applications," Multibody System Dynamics, 15 (3), pp. 241-262.

[18] Cheng, H., and Sandu, A., 2009, "Efficient Uncertainty Quantification with the Polynomial Chaos Method for Stiff Systems," Mathematics and Computers in Simulation, 79 (11), pp. 3278-3295.

[19] Xiu, D., and Hesthaven, J. S., 2005, "High-Order Collocation Methods for Differential Equations with Random Inputs," SIAM Journal on Scientific Computing, 27 (3), pp. 1118-1139.

[20] Xiu, D., 2007, "Efficient Collocational Approach for Parametric Uncertainty Analysis," Communications in Computational Physics, 2 (2), pp. 293-309.

[21] Xiu, D., 2009, "Fast Numerical Methods for Stochastic Computations: A Review," Communications in Computational Physics, 5 (2-4), pp. 242-272.

[22] Erdmann, M., 1984, "On Motion Planning with Uncertainty," Masters thesis, Massachusetts Institute of Technology, Boston.

[23] Barraquand, J., and Ferbach, P., 1995, "Motion Planning with Uncertainty: The Information Space Approach," International Conference on Robotics and Automation, 2, pp. 1341-1348.

[24] Park, W., Liu, Y., Zhou, Y., Moses, M., and Chirikjian, G., 2008, "Kinematic State Estimation and Motion Planning for Stochastic Nonholonomic Systems Using the Exponential Map," Robotica, 26 (04), pp. 419-434.

[25] Kewlani, G., Ishigami, G., and Iagnemma, K., 2009, "Stochastic Mobility-Based Path Planning in Uncertain Environments," pp. 1183-1189.