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KINEMATIC ANALYSIS OF A NOVEL RIMLESS WHEEL WITH INDEPENDENTLY ACTUATED SPOKES

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ABSTRACT

One of the major limitations of mobile robots for unstructured environments is their lack of general mobility. Wheeled, treaded, and legged robots each have their advantages and disadvantages, but they all lack the flexibility to be able to cope with a wide range of terrain. The actuated spoke wheel concept is presented in this paper as an alternative locomotive method that allows multiple modes of motion, which give it the ability to stride quickly using one contact point per wheel, walk with static stability with two contact points per wheel, or assume a stable stance using three contact points per wheel. This paper presents the preliminary kinematic analyses of the actuated spoke wheel with no-slip constraints at the ground contacts for a robot using a two actuated spoke wheel configuration. Straight-line motion and considerations for turning are discussed for the one- and two-point contact schemes followed by recommendations for future study.

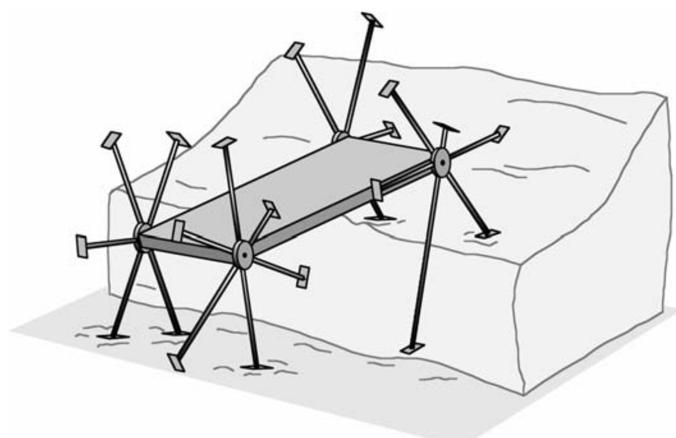

Figure 1. CONCEPTUAL SKETCH OF THE IMPASS-BASED ROBOT WITH FOUR ACTUATED SPOKE WHEELS

INTRODUCTION

Robot mobility is an area in need of much improvement, as today's robots are often limited by their lack of general mobility in unstructured environments [1]. Specialized robots have been designed for limited and specific tasks, but their mobility is not yet robust enough to handle varying terrain. Wheeled robots often have high efficiency and speed, but tend to be limited to relatively smooth terrain. Legged robots are adaptable and have good mobility on rough terrain; however, the main disadvantage of legged mobile robots is that the complexity of the leg usually necessitates a slow and inefficient mechanism [2].

The locomotive limitations of these two main types of mobile robots are currently countered in research by developing hybrid robots that add mechanisms to wheeled vehicles to give them improved mobility, such as the robot, Shrimp, developed by EPFL [3]. This robot has six motorized wheels and uses a combination of actuation and passive mechanisms to raise and lower its wheels to climb objects up to twice the wheel diameter. Improvements to legged mobile robots look to improve the efficiency of the legged design, such as RHex, developed in part

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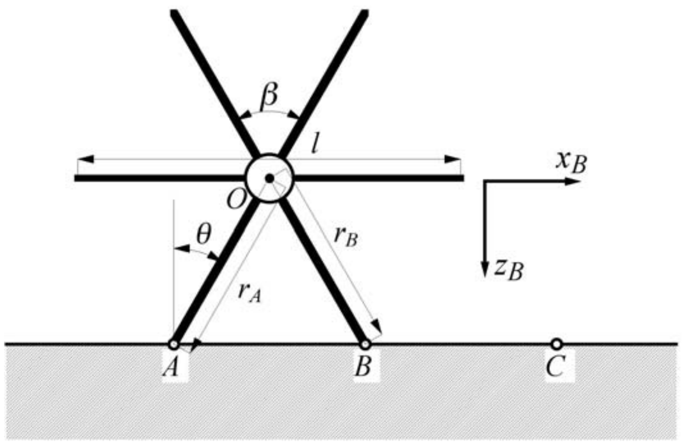

Figure 2. THE COORDINATE SYSTEM FOR THE PLANAR ACTUATED SPOKE WHEEL

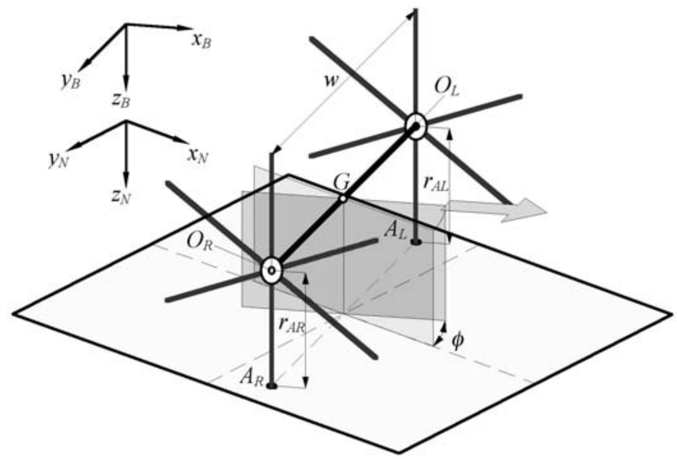

Figure 3. THE COORDINATES USED FOR THE DERIVATION OF THE KINEMATIC EQUATIONS

at the University of Michigan. RHex uses compliant legs in a hexapod configuration where each leg rotates full circle to walk a tripod gait [4, 5].

This paper introduces the rimless wheel with multiple, independently actuated spokes as a novel concept for creating a series of hybrid mobile robots with robust mobility that includes the benefits of both legged and wheeled mobile robots. The end goal of this research is to use the actuated spoke wheel as the basis for the Intelligent Mobility Platform with Active Spoke System (IMPASS). Figure 1 shows conceptual ideas for the robust locomotion available to a robot using the IMPASS concept.

The idea for the actuated spoke wheel is based on the passive rimless wheel, which has been studied for its application to the study of the human gait [6]. Variants of this idea include the single degree of freedom expanding spoked wheel reported by Yan and Agrawal [7, 8]. The advantage of our novel rimless wheel with multiple degrees of freedom presented here lies in its ability to move using several different modes of locomotion, so that it can adapt to its terrain as needed.

This paper will discuss the preliminary kinematic analyses for the one-, two-, and three-point contact per wheel schemes for a robot using two actuated spoke wheels over flat terrain. Straight-line and turning motion will be described for the one- and two-point contact schemes. The usefulness of these modes of locomotion will be discussed with some possible uses for each, followed by conclusions and discussion for future work.

MODEL OF THE ACTUATED SPOKE WHEEL SYSTEM

The development of the kinematic models for the actuated spoke wheel is based on a rimless wheel with three linearly actuated spokes that pass through the axis of the wheel in parallel planes, providing six effective spokes, as shown in Figure 2. The angle between the spokes, β, is fixed at 60°. Six effective spokes per wheel were chosen as a balance between the requirements for acceptable mobility and the increasing mechanical complexity that comes with adding additional spokes. Having the spokes pass through the axis of the wheel allows the number of actuators for the spokes to be reduced by half; thus with one additional actuator for the rotation of the wheel, only four actuators are needed for one, actuated spoke wheel with six effective spokes. Since three spokes can be independently actuated at a time, it becomes clear that it would be possible for the actuated spoke wheel to have one, two, or three contact points with the ground, with each of these modes of locomotion having very different mobility characteristics. Each contact point with the ground is considered to have an imposed no-slip condition. The preliminary analyses presented here will consider motion over flat terrain only.

ONE-POINT CONTACT SCHEME

Kinematics

The coordinate system for the model is developed based on the SAE J670e convention in which the x-axis is along the positive direction of travel, and the z-axis is oriented such that forces from the spokes to the ground are positive, as shown in Figure 2 and Figure 3. The ground is represented by the inertially fixed reference frame, $N\{x_N, y_N, z_N\}$, and the robot travels along a path frame, $P\{x_P, y_P, z_P\}$, that is rotated from the N-frame by angle ϕ about the z_N axis. A body fixed frame, $B\{x_B, y_B, z_B\}$, is created by choosing different spoke lengths ($r_{A_R} \neq r_{A_L}$), which will cause the robot to roll through an angle ψ about the x_P axis. Finally, a wheel fixed frame, $W\{x_W, y_W, z_W\}$, is created by the actuated spoke wheel pitching through an angle, θ, relative to the body about the y_B axis.

The center of the wheel is defined by point O and the wheel's contact points with the ground are labeled sequentially along the x_B axis as A, B, and C for cases of one-, two-, and three-point contact per wheel respectively, as shown in Figure 2. The length of a spoke (stroke) from a ground contact point to the center of the axle is denoted as r with the subscript of that contact point. In further discussion, these variables have a subscript of "R" or "L" to indicate reference to a particular wheel, with the wheel on the positive y_B axis considered the right wheel. When motion is considered over several steps, as in Figure 6, a subscript is used to indicate the designation of that contact point in reference to a particular step. For example, in the one-point contact case there is an instant where there are two points in contact with the ground as the wheel transitions from one step to the next. The rear most point would be A_i and the forward most point would be B_i. After the transition is complete, the forward most point for the previous step become the rear most point for the current step, and thereby become A_{i+1}.

The configuration of the robot used for this analysis consists of a robot that has two actuated spoke wheels in parallel planes, which are separated by a normal distance, w, with point G at the center of the axle connecting the wheels. The center of mass of the robot is considered here to coincide with point G. The actuated spoke wheels considered here are driven by a solid axle, so that the two actuated spoke wheels are always in phase ($\theta_R = \theta_L$).

The kinematic velocity equations are created by finding the velocity of point G relative to the ground contact of the right wheel, A_R which is a fixed point in the N-frame. The location of point G is then

$$\overrightarrow{A_R G} = -r_{A_R}\vec{z}_W - w/2\vec{y}_B \tag{1}$$

Taking the time derivative of this position vector gives the equations for the velocity of the center of the axle. Recognizing that the pitch angle, ψ, is a function of the spoke lengths r_{A_R} and r_{A_L} through the relationship

$$\psi = \sin^{-1}\left(\frac{r_{A_L} - r_{A_R}}{w}\right) \tag{2}$$

allows one to substitute to remove ψ from the kinematic equations. The constraints caused by the no-slip conditions at the two ground contact points (one for each wheel) ensure that the spokes actuate at the same rate, maintain that the heading angle cannot change during the course of a step, and that the velocity of the robot is constrained to the current heading angle. These constraints limit the motion of the actuated spoke wheel to a vertical plane over the course of a step. A planar mobility analysis can be performed on the mechanism that results from these constraints using Grubler's equation [9], which shows that the robot with two actuated spoke wheels with one contact per wheel has two degrees of freedom: it can pivot about the line, $\overline{A_R A_L}$, that passes through the ground contact points, and it can linearly actuate its spokes independent of the pivoting motion.

A set of differential kinematic equations can be developed using the three equations that result from taking the time derivative of Equation 1 and the equations that result from the constraints above [10]. The two degrees of freedom allow two specified inputs. The complete kinematic differential equations are too long to be listed here, but interested readers are encouraged to contact the authors for more information. In summary, there are seven states given by the three translational velocities of point G in the N frame, $\dot{x}$, $\dot{y}$, and $\dot{z}$, two linear velocities of the spokes, $\dot{r}_{A_R}$ and $\dot{r}_{A_L}$, and two rotational velocities given by the change in heading angle, $\dot{\phi}$, and the change of the wheel angle $\dot{\theta}$. From the mobility analysis, the constraint equations are given as

$$\dot{\phi} = 0 \tag{3}$$

$$\dot{r}_{A_R} = \dot{r}_{A_L} \tag{4}$$

This analysis will consider motion which specifies a vertical speed (u_z) and a longitudinal speed(u_x) along the current heading angle. The resulting input equations are

$$\dot{z} = u_z \tag{5}$$

$$\dot{x} = u_x \cos\phi \tag{6}$$

These equations allow the motion of the robot to be determined for a given set of input speeds u_z and u_x. The arbitrary nature of

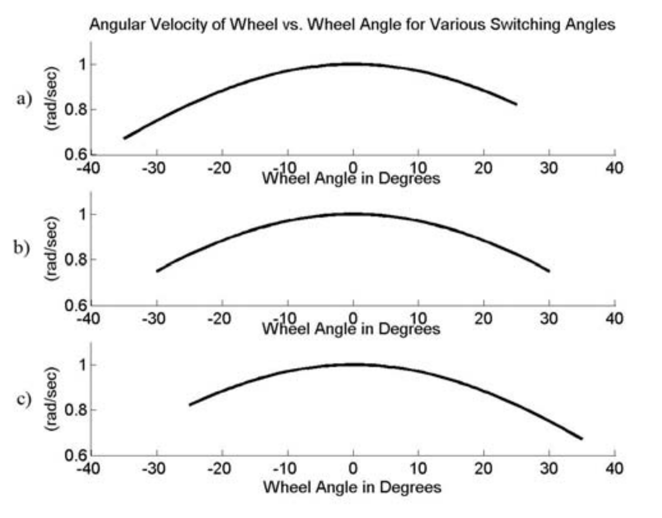

Figure 4. PLOTS OF THE ANGULAR VELOCITY OF THE WHEEL FOR SWITCHING ANGLE, θ_s, of a) 25° b) 30° c) 35°

this result highlights the flexibility of the design of the actuated spoke wheel concept.

Straight-Line Motion

The analysis of straight-line motion for a one-point contact scheme in this paper will consider the case in which the robot moves at constant speed parallel to the ground ($u_z = 0$ and $u_x = constant$). This type of locomotion is of interest because it removes the need to raise and lower the center of gravity of the robot for every step to conserve power which allows the robot to cover smooth terrain quickly without changing its potential energy, making the overall motion generated by the actuated spoke wheel comparable to the motion from a traditional wheel for smooth terrain. Since it is possible to independently actuate the spokes used for the current step and those to be used in the next step, it is possible to select the wheel angle, θ_s, at which the robot will switch contact points. To determine the best angle at which to switch contact points, the input rotational and linear velocities, $\dot{\theta}$ and $\dot{r}_A$, respectively, were plotted using MATLAB over the course of a step for a number of switching angles. These plots show that while the input from the spoke actuators remains nearly linear for the different cases, the rotational input from the wheel actuator are not. The required rotational input would be discontinuous from one step to the next for all switching angles except θ_s=30°. There would be discontinuities for the linear actuation of the spokes as well, but that would not present any difficulty for the motion, as the robot switches linear actuators for each step. Figure 4 shows the angular velocity of the wheel for different switching angles. This switching angle of 30° represents the instant during the step at which switching would occur with the spokes in an isosceles triangle configuration. Using this switching angle, choosing a height at which to keep the robot will enforce a step length. The robot in this configuration would be able to maintain a constant height of any positive value up to $\frac{\sqrt{3}}{2}l$, at which point the spokes are fully extended and the robot would be taking a step of length l.

Moving while keeping the axle of the robot at a constant height has a number of uses. Figure 5 shows the motion for the one-point contact scheme maintaining a constant height. This can be used when the robot must try to maintain a smooth motion, such as when carrying a sensitive payload, or to cover even terrain quickly and efficiently. This motion of moving at a constant height is a beneficial motion scheme, but it is only one of many motions allowed by the one-point contact scheme, as the inputs u_z and u_x are arbitrary. The ability of the robot to adjust its height, and thereby adjust its step length, allows it to move in a manner best suited to fit the situation.

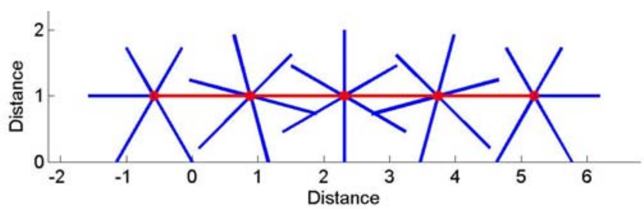

Figure 5. MOTION USING A ONE-POINT CONTACT SCHEME MAINTAINING CONSTANT HEIGHT

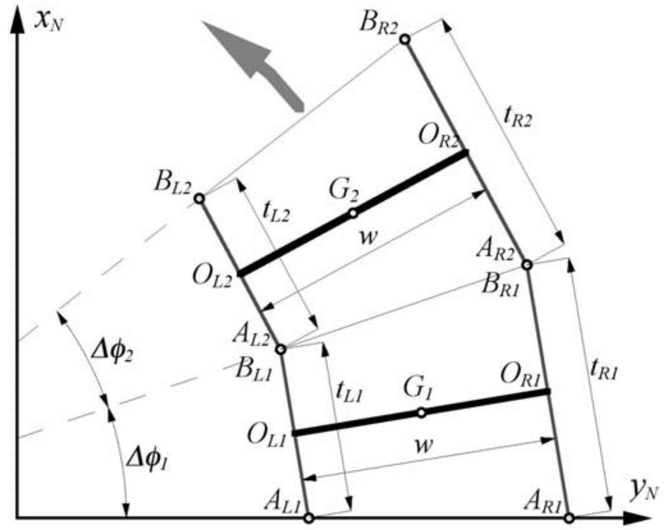

Figure 6. DISCRETE TURNING FOR THE ONE-POINT CONTACT SCHEME

Turning

Since the robot using actuated spoke wheels does not have a continuous contact path with the ground, but rather has a discrete number of contact points, it is not capable of continuously variable turning. Instead of turning by differential steering as is common in robots with two traditional wheels, or Ackerman steering as found in automobiles, turning the two actuated spoke wheel robot using a one-point contact scheme can be accomplished by choosing different spoke lengths for each side of the robot such that the axle has different normal distances from the ground at each end. Choosing different spoke lengths changes the effective radii of the wheels independently.

Since axle height and step length are related in one-point contact, the robot will take a longer step with one actuated spoke wheel than with the other. With the heading angle defined by the direction of the line $\overline{A_R A_L}$ which passes through the two ground contact points, the heading angle from one step to the next will change by making one side's step longer than the other, as shown in Figure 6, making the robot turn in a discrete fashion in an amount related to the difference in step lengths. Each step taken with unequal lengths introduces a change in the heading angle, denoted by $\Delta\phi$ and this relationship is given by

$$\Delta\phi_i = \tan^{-1}\left(\frac{t_{Li}-t_{Ri}}{w}\right) \quad (7)$$

where t_R and t_L are the step lengths of the right and left actuated spoke wheels, respectively.

TWO-POINT CONTACT SCHEME

Kinematics

As with the one-point contact schemes, motion using a two point contact scheme is constrained to a plane by the no-slip conditions at the ground contacts. A mobility analysis of the robot in the two point contact stance using Gruber's equation indicates one degree of freedom. With a no-slip condition at both contact points for each wheel, the distance between the contacts is fixed, and since the angle between the spokes in contact with the ground is fixed, it is possible to derive the relationship for the position of the axle of the robot as a function of the wheel angle, θ, using the law of sines.

Figure 2 shows a the robot in a two-point contact step. It can be shown that the relationships for the length from the rear contact point, A, to the axle, r_A, and for the length from the forward contact point, B, to the axle, r_B, are given by

$$r_A = t\frac{\sin(\pi/2+\theta-\beta)}{\sin\beta} \quad (8)$$

$$r_B = t\frac{\sin(\pi/2-\theta)}{\sin\beta} \quad (9)$$

where t is the ratio of the step distance, $\overline{AB}$, to the total length of the spoke, l. The flexibility of the design allows for t to be chosen from for any positive value up to $\frac{\sqrt{3}}{2}$, at which point the spoke is fully extended during the step. Once a step length is chosen, the robot will move along a specified path as a function of the wheel angle.

Straight-Line Motion

For straight-line motion, the robot would again take steps of equal lengths with both wheels. The resulting motion is plotted using MATLAB for several steps of constant length in Figure 7. The robot can begin in a three-point contact position and start walking by lifting its rear contact point, such that the step begins with axle directly over one contact point (θ=0°). The axle of the robot will travel through an arcing path until it is directly over the next contact point ($\theta = 60°$), at which point, it will have three contact points again. It will lift its rear-most contact point and repeat for the next step. Using this scheme, it is possible to prevent the center of gravity of the robot from moving outside the area between the contact points, making walking in this method statically stable.

The prominent feature of this motion is the repeated arcing pattern of axle's path. This is analogous to the motion of the center of gravity of a passive rimless wheel, which is often used to approximate bipedal walking motion. While the motion of the actuated spoke wheel is not constrained to a circular arc as for the case of the passive rimless wheel, it does provide a viable scheme for statically stable walking with as few as two actuated spoke wheels. This represents a significant achievement in simplicity of design over traditional, legged mobile robots, which require multiple legs with multiple degrees of freedom for static stability during walking.

Since there are more contact points with the ground and more actuators than degrees of freedom, this scheme could be used for carrying a heavy payload, by distributing the load carried over multiple actuators. It has the advantage of traditional legged robots in that it would be capable of stepping over obstacles, but since the actuated spoke wheel rotates continuously, it is unhindered by the concern of tripping due to catching the foot during the swing phase of normal walking.

Turning

Turning using a two-point contact scheme would require a similar approach of taking steps of different lengths with the left and right wheels. Referring back to Equations 8 and 9, it can be seen that the linear speed of the spoke actuators is proportional to the length of the step taken. Since turning would require taking steps of different lengths, it would also require that the left and right spokes actuate at different speeds. However, that motion cannot occur without slipping of the ground contact points. Skidding conditions have not been considered at this time, but are listed in the conclusion as a topic for future study.

Three-Point Contact

A mobility analysis of the three-point contact mode (shown in the left- and right-most positions of Figure 7) shows that there are zero degrees of freedom, and as such, this is not a scheme

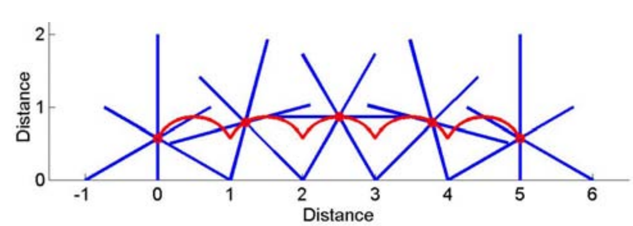

Figure 7. PATH OF THE AXLE FOR MOTION USING THE TWO-POINT CONTACT SCHEME

for motion. However, the three-point contact scheme is still of use to the actuated spoke robot. The advantage of this mode is that the robot can take a very wide stance on terrain for improved stability. As with the other schemes, the length of the stance is related to the height of the robot. The robot would be able to assume a three-point contact position at essentially zero ground clearance to prevent rolling, or it can stand at half the total spoke length ($l/2$) with a stance that is $\sqrt{3}l$.

This is a statically stable position which can be used for bracing the robot at rest. This type of position could be useful for additional stability on uneven terrain for a mobile robot performing stationary manipulation tasks, such as digging, drilling, or collecting rocks for samples. This stable position itself is not unique, and since each of the spokes are capable of being adjusted independently, the left and right wheels can be adjusted independently to brace the robot in a stance best suited for the terrain.

CONCLUSION AND FUTURE WORK

In this paper, the actuated spoke wheel is presented as a novel concept for robot locomotion. A robot using two actuated spoke wheels is analyzed on flat terrain using a one-, two-, and three-point contact per wheel scheme. These modes are analyzed to show the motions available to a robot using the actuated spoke wheel when constrained to non-slipping contacts with the ground. It is shown that the one-point contact mode has two degrees of freedom and that the motion output can be arbitrarily selected. This mode would allow for moving while maintaining a constant height for the center of mass, which is analyzed here. Turning for this mode is shown to occur discretely by changing the heading angle for every step by taking steps of different lengths with the right and left wheels. The two-point contact mode is shown to have one degree of freedom, and that by choosing a step length, the path of the center of the axle in the sagittal plane is determined as a function of the wheel angle, θ. This mode of locomotion allows for statically stable walking with only two wheels, and could be used for carrying heavy payloads. The three-point contact scheme is shown to have zero degrees of freedom, but would allow for additional stability during stationary tasks by letting the robot assume a wide stance.

Future work will focus on investigating different motion schemes utilizing individual spoke actuation to improved mobility. Further kinematic work needs to be performed to understand the motion of the robot as it transitions from one motion scheme to another, to study the motion over uneven terrain, and to determine the functionality of the actuated spoke wheeled robot in other configurations, such as allowing the left and right wheels to rotate independently, or a configuration with four wheels as shown in Figure 1. Work will continue into developing algorithms and strategies for intelligent motion planning and coordination of the active spokes for climbing over obstacles. Other work will include dynamic analysis and a study of energetics of the various actuated spoke wheel configurations.

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