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INSTANTANEOUS KINEMATICS AND SINGULARITY ANALYSIS OF A NOVEL THREE-LEGGED MOBILE ROBOT WITH ACTIVE S-R-R-R LEGS

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ABSTRACT

STriDER (Self-excited Tripedal Dynamic Experimental Robot) is a unique three-legged walking robot that utilizes its innovative tripedal gait to walk. Previous work on the kinematic analysis of STriDER mainly focused on solving the forward and inverse displacement problems. As a continuation, this paper addresses the instantaneous kinematics and singularity analysis. The kinematic configuration of STriDER is modeled as a three-legged in-parallel manipulator when all three feet of the robot are in contact with the ground without slipping. The results obtained from this study can be implemented to the velocity control and the resistance of disturbance forces, thus improving the motion accuracy and stability of the robot. By using *screw theory*, the screw-based Jacobian matrices of the manipulator can be derived since the forward displacement problems have already been solved. Based on these Jacobian matrices, the transformation equations from the active joint rates to the velocities of the body and vice versa are derived. Then, a complete investigation on the identification and elimination of singularities is presented. Unlike serial manipulators, in-parallel manipulators have two types of singularities, i.e., forward and inverse singularities. The inverse singularities are identified by checking the singular configurations of individual legs and the determinant of the inverse Jacobian matrix. By using *Grassmann line geometry*, the analytical conditions under which the forward singularities occur are obtained. A study on each case of these singular configurations shows that the redundant-actuation scheme of the active joints can effectively eliminate forward singularities.

1. INTRODUCTION

STriDER, as shown in FIG. 1, is an innovative three-legged walking machine that is under developing at RoMeLa: Robotics & Mechanisms Laboratory in Virginia Tech. Each leg of the robot has features three active revolute joints, located at the hip rotator, the hip flexure and the knee respectively.

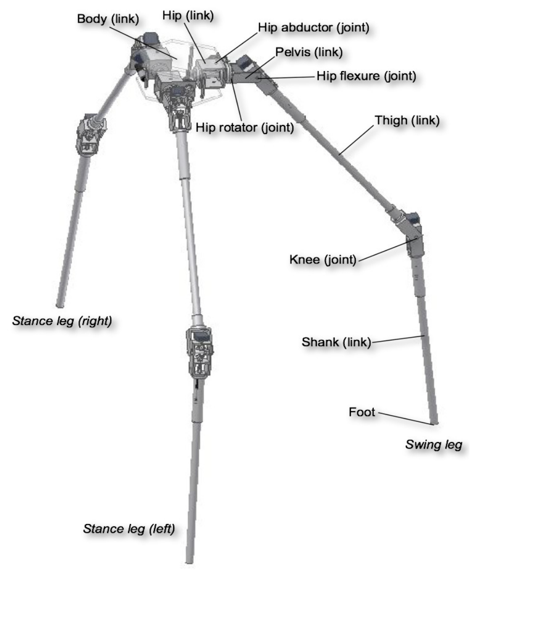

FIG. 1. STriDER (SELF-EXCITED TRIPEDAL DYNAMIC EXPERIMENTAL ROBOT)

1.1 SCHEME OF LOCOMOTION

The novel tripedal gait (patent pending) of this robot can be simply implemented for a single step. During a step, two legs act as stance legs while the third acts as a swing leg. STriDER first begins with a stable tripod stance, then the hip links are oriented to push the center of gravity forward by

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aligning the stance legs' pelvis links. As the body of the robot falls forward, the swing leg naturally swings in between the two stance legs and catches the fall. As the robot takes a step, the body needs to rotate 180° to prevent the legs from tangling up. Once all three legs are in contact with the ground, the robot regains its stability and the posture of the robot is reset in preparation for the next step. A video of this dynamic gait can be seen in [1]. By changing the sequence of choice of the swing leg, the tripedal gait can turn the robot to different directions.

1.2 PREVIOUS RESEARCH

Previous research on STriDER mainly included [2],[3] and [4]. In [2], Heaston developed the preliminary design optimization of the link parameters such as link length, inertia and etc., based on the accomplishment of a single dynamic step. Lahr then designed a joint aligning mechanism to replace the three hip abductors in the body which are shown in FIG.1 (the transmission mechanisms is not shown in this figure), thus reducing the weight of the body dramatically [3].

The forward and inverse displacement analysis of STriDER when its all three feet are in contact with the ground without slipping was developed in [4]. This analysis is quite necessary because the robot is expected to deploy sensors such as cameras in unstructured environment. To control the change of body's posture when all three feet are on the ground requires a complete study on the kinematics of its forward and inverse displacement. In this work, STriDER was modeled as a three-legged in-parallel manipulator. The inverse displacement problems were solved with geometrical method. The forward displacement analysis for STriDER was solved with loop-closure equations based on geometric constraints and the intersection of the 3D loci of the feet. Results showed that, for redundant sensing with displacement information from nine, eight, and seven joint angle sensors, analytical solutions can be obtained. Particularly, when all nine joint angles are known, there is only one analytical solution. For non redundant sensing cases, or with displacement information from six joint angle sensors, analytical solutions can be obtained when the displacement information is known from non-equally distributed joint angle sensors between the three legs. On the other hand, for equally distributed joint angle sensors, a 16th-order polynomial with respect to one variable can be derived to solve the problem.

Three prototypes of STriDER developed in RoMeLa are displayed in FIG.3 as follows.

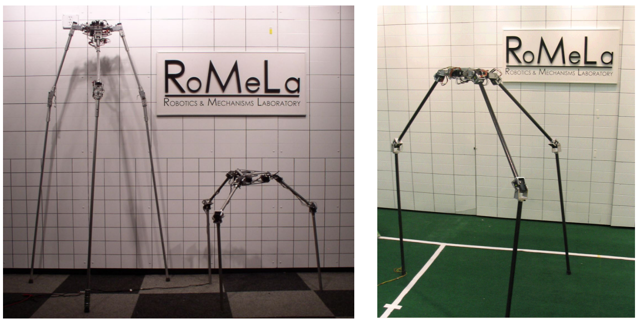


FIG. 3. STriDER PROTOTYPES

1．3 CURRENT RESEARCH

The focuses of the study in this paper are instantaneous kinematics and singularity analysis. By establishing the Jacobian matrices, the implementation of the velocity control of the robot's body is theoretically available. Identification and elimination of the singular configurations can improve motion accuracy and stability of STriDER when it performs surveillance operations in the fields. Again, STriDER is modeled as a three-legged in-parallel manipulator with Spherical-Revolute-Revolute-Revolute (S-R-R-R) joints in each leg as shown in FIG.4. Note that, each of the contact point of the foot with the ground is modeled as a spherical joint with 3 passive d.o.f and the robot system has 6 d.o.f. in 3D space. The triangle formed by three feet on the ground is considered as the "virtual base" of this manipulator. Please note that, since STriDER is inherently a walking robot, the shape of the base triangle is not fixed but dependent on the potions of the feet.

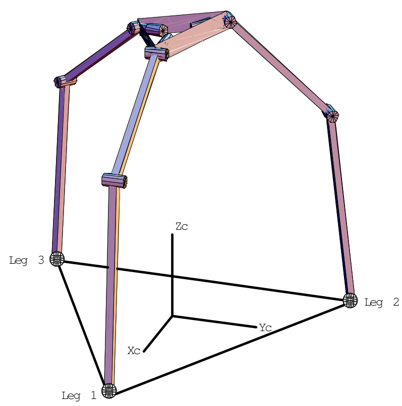


FIG. 4. 3 S-R-R-R IN-PARALLEL MANIPULATOR

The structure of this paper is listed as follows. In Section 2, the kinematic configuration and definitions of coordinate systems are briefly reviewed firstly. Then the basic concepts of *screw theory* are introduced and screw-based Jacobian matrices are developed, which relate the joint rates to the velocity of the body. In Section 3, *Grassmann line geometry* is reviewed and the inverse and forward singular configurations are identified. Section 4 mainly shows the examples of the forward singular configurations. Conclusion and future research are discussed in Section 5.

2. INSTANTANEOUS KINEMATICS

Instantaneous kinematics, also known as Jacobian analysis, relates the velocity space of the body of the in-parallel manipulator to the internal joint rates space. It also provides insights into the singular configurations of the manipulator system. In Section 2.1, the definitions of coordinate systems in [4] are briefly reviewed. The setup of the reference coordinate frames based on standard DH method is still used in this work to derive the screw coordinates of active joints.

2.1 KINEMATIC CONFIGURATION OF STriDER

The definition of coordinate systems for each leg is shown in FIG. 5. The subscript i denotes the leg number (i.e. i =1, 2, 3) in the coordinate frames, links, and joint labels.

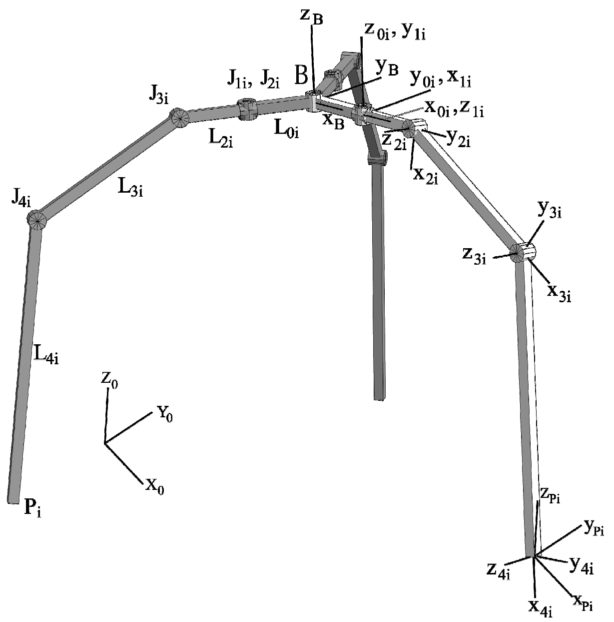


FIG. 5. COORDINATE FRAME AND JOINT DEFINITIONS

Table 1 lists the nomenclature used to define the coordinate frames, joint and links. A global coordinate system, $\{X_0, Y_0, Z_0\}$ and the body coordinate frame $\{x_B, y_B, z_B\}$ are shown in FIG. 5. Each leg is separated by 120 degrees; leg 1, leg 2 and leg 3 are 0 degrees, 120 degrees, and 240 degrees from the positive x_B axis, respectively. Each leg includes four joints, J_{1i}, J_{2i}, J_{3i}, and J_{4i}. Note that three hip abductor joints, J_{1i}, which control the stance legs' rotator joints to align during a step, is replaced with a joint aligning mechanism driven by only one motor in [3]. Thus J_{1i} is not treated as an active joint in this paper and each leg has 3 active joints at most. Two coordinate frames $\{x_{4i}, y_{4i}, z_{4i}\}$ and $\{x_{Pi}, y_{Pi}, z_{Pi}\}$ are established at each foot. The three unit vectors in frame of $\{x_{Pi}, y_{Pi}, z_{Pi}\}$ are defined to be parallel to the global vector units. The foot contact points denoted by P_i are treated as spherical joints during analysis and $\{x_{4i}, y_{4i}, z_{4i}\}$ relates to $\{x_{Pi}, y_{Pi}, z_{Pi}\}$ with three Euler angles.

The coordinate systems are defined following the standard DH convention [5] and the link parameters are listed in Table 2 where, j is the link number, (j = 1,2,3,4), i is the leg number (i = 1, 2, 3), a_{ji} equals the distance along x_{ji} from J_{ji} to the intersection of the x_{ji} and $z_{(j-1)i}$ axes, d_{ji} is the distance along $z_{(j-1)i}$ from $J_{(j-1)i}$ to the intersection of the x_{ji} and $z_{(j-1)i}$ axes, α_{ji} is the angle between $z_{(j-1)i}$ and z_{ji} measured about x_{ji}, and Θ_{ji} is the angle between the $x_{(j-1)i}$ and x_{ji} measured about $z_{(j-1)i}$. Also, when all Θ_{ji} are equal to zero, the legs form a right angle between L_{2i} and L_{3i}.

TABLE 1. NOMENCLATURE

Nomenclature	
i:	Leg number (i=1,2,3)
$\{X_0, Y_0, Z_0\}$:	Global fixed coordinate system
$\{x_B, y_B, z_B\}$:	Body center coordinate system
J_{1i}:	Hip abductor joint for leg i
J_{2i}:	Hip rotator joint for leg i
J_{3i}:	Hip flexure joint for leg i
J_{4i}:	Knee joint for leg i
P_i:	Foot contact point for leg i
L_{0i}:	Body link for leg i
L_{1i}:	Hip link for leg i (length =0)
L_{2i}:	Pelvis link for leg i
L_{3i}:	Thigh link for leg i
L_{4i}:	Shank link for leg i

TABLE 2. LINK PARAMETERS

Link	a_{ji}	α_{ji}	d_{ji}	Θ_{ji}
1	L_{1i}=0	90°	0	Θ_{1i}+90°
2	0	0	L_{2i}	Θ_{2i}-90°
3	L_{3i}	0	0	Θ_{3i}
4	L_{4i}	0	0	Θ_{4i}

2.2 INTRODUCTION TO SCREW AND RECIPROCAL SCREW

Screw theory, as a powerful tool in kinematics analysis of mechanisms, is extensively treated in [6] [7] and [8]. Both finite and infinitesimal displacement of a rigid body can conveniently be expressed as a rotation about a unique axis and a translation along the same axis. This combined motion is called *twist*, and the unique axis is called a *screw axis* of the displacement. Due to the duality of statics and instantaneous kinematics, a similar concept can be defined. Any system of forces and couples acting on a rigid body can be reduced to a resultant force and a couple acting on the same axis. The force and couple combination is called a *wrench.* The *twist* and the *wrench* can be denoted as six-dimensional vectors called *screws* as follows:

$$\mathbf{T}=[\mathbf{w},\mathbf{q}\times\mathbf{w}+\mathbf{v}]^{\mathrm{T}};\quad \mathbf{W}=[\mathbf{f},\mathbf{p}\times\mathbf{f}+\mathbf{t}]^{\mathrm{T}} \qquad (2.1)$$

where **w** is the angular velocity and **v** is the linear velocity of point q, represented by vector **q**; **f** and **t** are the force and couple of the wrench acting on point p, represented by vector **p**. The pitch h of **T** is defined as $h=|\mathbf{v}|/|\mathbf{w}|$ and the pitch h' of **W** is defined as $h'=|\mathbf{t}|/|\mathbf{f}|$. $|\mathbf{v}|,|\mathbf{w}|$ and $|\mathbf{t}|,|\mathbf{f}|$ are the magnitude of **v**, **w** and **t**, **f**, respectively.

In most robotics literatures, the term **$** is usually used to represent a screw for a joint. As for a revolute joint, h = 0, the unit joint screw $\hat{\$}$ can be expressed as:

$$\hat{\$}=\begin{bmatrix}\mathbf{s}\\ \mathbf{r}\times\mathbf{s}\end{bmatrix}=\begin{bmatrix}S_1 & S_2 & S_3 & S_4 & S_5 & S_6\end{bmatrix}^{\mathrm{T}} \qquad (2.2)$$

As for a prismatic joint, h = ∞, the unit joint screw can be expressed as:

$$\hat{\$} = \begin{bmatrix} 0 \\ \mathbf{s} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & S_1 & S_2 & S_3 \end{bmatrix}^{\mathrm{T}} \quad (2.3)$$

where **s** represents the directional unit vector of the joint axis ans **r** is the position vector of any point on the joint axis. Taking Eq.(2.2) as a example, the *transpose of a screw* is defined as:

$$\hat{\$}^{\mathrm{T}} = \begin{bmatrix} S_4 & S_5 & S_6 & S_1 & S_2 & S_3 \end{bmatrix} \quad (2.4)$$

A *wrench* and a *twist* are said to be *reciprocal* if the virtual work of the wrench on the twist is zero. The basic reciprocal condition is as follows:

$$\begin{aligned} \$_r{}^{\mathrm{T}}\$ &= S_1 S_{r4} + S_2 S_{r5} + S_3 S_{r6} + S_4 S_{r2} + S_6 S_{r3} = 0 \\ \text{or } & (h + h')\cos\lambda - R\sin\lambda = 0 \end{aligned} \quad (2.5)$$

where R is the shortest distance between the screw axes and λ is the angle between them. An investigation of reciprocal screws based on Eq. (2.5) is provided in [7]. The reciprocal screw systems of 1 d.o.f. joints (revolute, prismatic), 2 d.o.f joint (universal) and 3 d.o.f joint (spherical) are addressed by Tsai in [9]. In the following sections, the theory of reciprocal screws is extensively used to in the process of the development of Jacobian matrices and singularity analysis.

2.3 THE DERIVATION OF JACOBIAN MATRICES

The instantaneous twist $\$_B$, of the body of the in-parallel manipulator can be expressed as a linear combination of l instantaneous joint twists, which was presented in [10].

$$\$_{\mathrm{B}} = \sum_{j=1}^{l} \dot{q}_{j,i} \hat{\$}_{j,i} \qquad ,\text{for } i = 1, 2, \ldots, m \quad (2.6)$$

where $\dot{q}_{j,i}$ and $\hat{\$}_{j,i}$ denote the magnitude and the unit screw associated with the jth joint of the ith leg, m is the number of legs of the parallel manipulator. Note that, instantaneous kinematic analysis must be developed after the forward displacement problems have been successfully solved. That is because in the derivation of Jacobian matrices, the screw coordinates in Eq.(2.6) must be known at any instant.

When Eq.(2.6) is assembled into matrix form, the screw-based Jacobian matrices can be derived. However the unactuated joint rates in Eq. (2.6) must be eliminated at first. This elimination can be accomplished using the theory of reciprocal screws. Assume that g actuated joints appear in leg i, for an actuated joint j of the g joints, a reciprocal screw is identified that is reciprocal to all the other joint screws in leg i, except for joint j. Take the reciprocal product of both sides of Eq. (2.6) with each reciprocal screw. Then g equations can be written which correspond to g actuated joints in leg i. Repeating this procedure for each of the m legs yields $n = m \times g$ linear equations which can be assembled in matrix form:

$$\mathbf{J}_x \$_{\mathrm{B}} = \mathbf{J}_q \dot{\mathbf{q}} \quad (2.7)$$

Thus, the Jacobian matrices associated with the forward kinematics $\mathbf{J}_x$ and the inverse kinematics $\mathbf{J}_q$ are derived. Both $\mathbf{J}_x$ and $\mathbf{J}_q$ are screw-based Jacobian matrices and the rows of $\mathbf{J}_x$ are actually the reciprocal screw identified for each actuated joint.

The identification of appropriate reciprocal screws is crucial for the development of screw-based Jacobian matrices of in-parallel manipulators. Since each reciprocal screw is chosen to be reciprocal to all the joint screws, except for just one of the actuated joint screws, $\mathbf{J}_q$ is greatly simplified into a diagonal matrix. Thus, the Jacobian matrices have more compact forms and the computation time of solving such a matrix equation will be much shorter.

As for the case of this three-legged S-R-R-R kinematic model of STriDER, the reciprocal screws associated with each active joint is much easier to identify. Due to the all-revolute joints in each leg of STriDER, all joint screws are zero-pitch screws that are the same as the *Plücker coordinates* of straight lines in 3D space. According to Eq. (2.5), finding a reciprocal screw to an active joint screw in a given leg is equivalent to finding a line that intersects (R = 0) all the other joint axes in this leg except the axis of the active joint. Note that a line that is parallel to another line ($\lambda = 0$) is also treated as "intersecting at infinity", because their reciprocal product is also zero by Eq.(2.5). FIG.6 is used to assist the identification of the reciprocal screws, which shows the joint screws and reciprocal screws in leg i. In this figure, $\mathbf{\$}_{1i}$, $\mathbf{\$}_{2i}$ and $\mathbf{\$}_{3i}$ are the three actively controlled joint screws in leg i. These three joint screws are equivalent to the axes of joint J_{2i}, J_{3i} and J_{4i} in leg i, i.e. the hip rotator joint, the hip flexure joint and the keen joint as in Table 1. There are also three passive joint screws $\mathbf{\$}_{4i}$, $\mathbf{\$}_{5i}$ and $\mathbf{\$}_{6i}$ at the foot the leg i. These three joints screws are orthogonal to each other and interesting at the foot contact point with the ground. Therefore, identification of the reciprocal screws to any screw of $\mathbf{\$}_{1i}$, $\mathbf{\$}_{2i}$ and $\mathbf{\$}_{3i}$ is developed by locating a line that intersects the rest two screws of $\mathbf{\$}_{1i}$, $\mathbf{\$}_{2i}$ and $\mathbf{\$}_{3i}$ and three passive joint screws $\mathbf{\$}_{4i}$, $\mathbf{\$}_{5i}$ and $\mathbf{\$}_{6i}$. These identified reciprocal screws are represented with red dash lines and denoted with $\mathbf{\$r}_{1i}$, $\mathbf{\$r}_{2i}$ and $\mathbf{\$r}_{3i}$ in FIG.6.

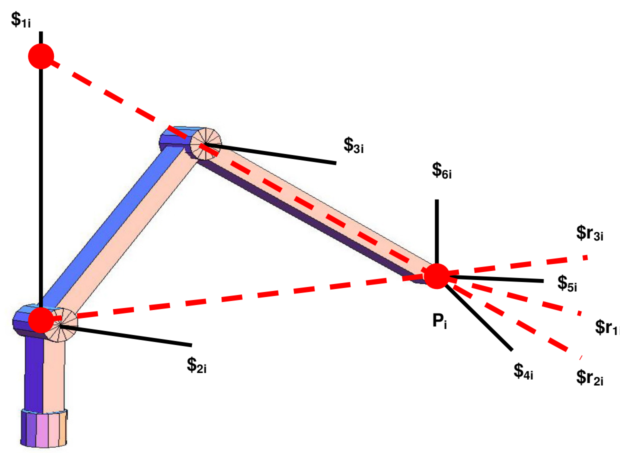


FIG.6. THE JOINT SCREWS AND RECIPROCAL SCREWS IN LEG I

$\$\mathbf{r}_{1i}$ is the reciprocal screw to $\$_{1i}$. It is a line that passes through the foot contact point $\mathbf{P}_i$ and parallel to $\$_{2i}$ and $\$_{3i}$. If one line is parallel to another line, the angle λ between the two parallel lines is zero, which means the Eq.(3.2) still holds true and parallel lines can be considered as intersecting at "infinity".

$\$\mathbf{r}_{2i}$ is the reciprocal screw to $\$_{2i}$. It passes $\mathbf{P}_i$ thus intersecting $\$_{4i}$, $\$_{5i}$ and $\$_{6i}$. By inspection, a line with the same direction as the shank line of STriDER's leg is the only candidate that intersects all joint axes except $\$_{2\mathbf{i}}$.

$\$\mathbf{r}_{3i}$ is the reciprocal screw to $\$_{3i}$. Since $\$_{1i}$ and $\$_{2i}$ intersects at the origin of joint 3, a line that connects the intersection point of $\$_{1i}$ and $\$_{2i}$ and the foot contact point is the reciprocal screw we are looking for.

As described in Section 2.1, two Cartesian coordinate systems $\{X_0,Y_0,Z_0\}$ and $\{x_B,y_B,z_B\}$ are attached to the ground and the body of STriDER, respectively. An instantaneous reference frame is set at the body center coordinate system $\{x_B,y_B,z_B\}$. Then, all joint screws and reciprocal screws are expressed with respect to the body frame.

The term $\hat{\$}_{ji}$ is used to represent any unit joint screw in any leg, with i = 1,2,3 (numbering of the legs) and j = 1,2,3,4,5,6 (numbering of the joints in a leg). Each unit screw has $\hat{\$}_{ji}$ two parts. The real unit is denoted with $\mathbf{s}_{ji}$ and the dual unit is denoted with $\mathbf{So}_{ji}$. A point on the axis of $\$_{ji}$ is represented with $\mathbf{r}_{ji}$ and often set at the origin of the joint coordinates. According to the definitions of revolute joint screws in Eq.(2.2), $\mathbf{So}_{ji} = \mathrm{r}_{ji} \times \mathbf{s}_{ji}$. Using the DH parameters described in Section 2.1 and listed in Table 2, the joint screws can be written as follows:

$$\begin{aligned}
\mathbf{s}_{1i} &= {}^{\mathrm{B}}\mathbf{R}_{0i}\,{}^{0i}\mathbf{R}_{1i}\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T \\
\mathbf{s}_{2i} &= {}^{\mathrm{B}}\mathbf{R}_{0i}\,{}^{0i}\mathbf{R}_{1i}\,{}^{1i}\mathbf{R}_{2i}\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T \\
\mathbf{s}_{3i} &= {}^{\mathrm{B}}\mathbf{R}_{0i}\,{}^{0i}\mathbf{R}_{1i}\,{}^{1i}\mathbf{R}_{2i}\,{}^{2i}\mathbf{R}_{3i}\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T \\
\mathbf{s}_{4i} &= \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T \\
\mathbf{s}_{5i} &= \begin{bmatrix}0 & 1 & 0\end{bmatrix}^T \\
\mathbf{s}_{6i} &= \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T
\end{aligned} \tag{2.8}$$

where matrix ${}^N\mathbf{R}_M$ is the 3 by 3 rotational part of the homogeneous transformation matrix ${}^N\mathbf{H}_M$ based on DH parameters, which transforms the homogeneous coordinates from frame M to frame N. The coordinate systems B, 0i, 1i, 2i, 3i and 4i with i = 1, 2, 3 in Eq.(2.8) are shown in FIG.5.

The point $\mathbf{r}_{ji}$ on the joint axes J_{ji} is set at the origin of the joint coordinate system for convenience. Homogeneous transformation matrices ${}^N\mathbf{H}_M$ are used to calculate the position vector.

$$\begin{aligned}
\begin{bmatrix}\mathbf{r}_{1i} \\ 1\end{bmatrix} &= {}^{\mathrm{B}}\mathbf{H}_{0i}\,{}^{0i}\mathbf{H}_{1i}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T \\
\begin{bmatrix}\mathbf{r}_{2i} \\ 1\end{bmatrix} &= {}^{\mathrm{B}}\mathbf{H}_{0i}\,{}^{0i}\mathbf{H}_{1i}\,{}^{1i}\mathbf{H}_{2i}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T \\
\begin{bmatrix}\mathbf{r}_{3i} \\ 1\end{bmatrix} &= {}^{\mathrm{B}}\mathbf{H}_{0i}\,{}^{0i}\mathbf{H}_{1i}\,{}^{1i}\mathbf{H}_{2i}\,{}^{2i}\mathbf{H}_{3i}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T \\
\begin{bmatrix}\mathbf{r}_{4i} \\ 1\end{bmatrix} &= {}^{\mathrm{B}}\mathbf{H}_{0i}\,{}^{0i}\mathbf{H}_{1i}\,{}^{1i}\mathbf{H}_{2i}\,{}^{2i}\mathbf{H}_{3i}\,{}^{3i}\mathbf{H}_{4i}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T
\end{aligned} \tag{2.9}$$

$$\mathbf{r}_{6i} = \mathbf{r}_{5i} = \mathbf{r}_{4i}$$

Again, with the assistance of transformation matrices ${}^N\mathbf{R}_M$ and ${}^N\mathbf{H}_M$, the unit reciprocal screws $\$\mathbf{r}_1$, $\$\mathbf{r}_2$ and $\$\mathbf{r}_3$ are calculated as follows:

$$\begin{aligned}
\mathbf{s}_{r,1i} &= \mathbf{s}_{2i} = \mathbf{s}_{3i} \\
\mathbf{r}_{r,1i} &= \mathbf{r}_{4i} = \mathbf{r}_{5i} = \mathbf{r}_{6i}
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\mathbf{s}_{r,2i} &= (\mathbf{r}_{4i} - \mathbf{r}_{3i}) / \|\mathbf{r}_{4i} - \mathbf{r}_{3i}\| \\
\mathbf{r}_{r,2i} &= \mathbf{r}_{4i} = \mathbf{r}_{5i} = \mathbf{r}_{6i}
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
\mathbf{s}_{r,3i} &= (\mathbf{r}_{4i} - \mathbf{r}_{2i}) / \|\mathbf{r}_{4i} - \mathbf{r}_{2i}\| \\
\mathbf{r}_{r,3i} &= \mathbf{r}_{4i} = \mathbf{r}_{5i} = \mathbf{r}_{6i}
\end{aligned} \tag{2.12}$$

Recall Eq.(3.3)

$$\$_{\mathrm{B}} = \sum_{j=1}^{l} \dot{q}_{j,i}\hat{\$}_{j,i} \qquad ,\text{for } i = 1, 2, \ldots, m$$

Substitute the joint screws obtained from Eq.(2.8) and Eq.(2.9) into Eq.(2.7) and take the orthogonal product of both sides of Eq.(2.7) with the reciprocal screws obtained from Eq.(2.10) to Eq.(2.12). Perform the operations for each of the three legs and assemble the equations in matrix form, we obtain

$$\mathbf{J}_x \$_B = \mathbf{J}_q \dot{\mathbf{q}} \tag{2.13}$$

where

$$\$_B = \begin{bmatrix} {}^B\boldsymbol{\omega}_n \\ {}^B\mathbf{v}_0 \end{bmatrix} = \begin{bmatrix} {}^B\omega_x & {}^B\omega_y & {}^B\omega_z & {}^Bv_{px} & {}^Bv_{py} & {}^Bv_{pz} \end{bmatrix}^T$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\theta}_{11} & \dot{\theta}_{21} & \dot{\theta}_{31} & \dot{\theta}_{12} & \dot{\theta}_{22} & \dot{\theta}_{32} & \dot{\theta}_{13} & \dot{\theta}_{23} & \dot{\theta}_{33} \end{bmatrix}^T$$

and

$$\mathbf{J}_x = \begin{bmatrix} \mathbf{So}_{r,11}{}^T & \mathbf{s}_{r,11}{}^T \\ \mathbf{So}_{r,21}{}^T & \mathbf{s}_{r,21}{}^T \\ \mathbf{So}_{r,31}{}^T & \mathbf{s}_{r,31}{}^T \\ \mathbf{So}_{r,12}{}^T & \mathbf{s}_{r,12}{}^T \\ \mathbf{So}_{r,22}{}^T & \mathbf{s}_{r,22}{}^T \\ \mathbf{So}_{r,32}{}^T & \mathbf{s}_{r,32}{}^T \\ \mathbf{So}_{r,13}{}^T & \mathbf{s}_{r,13}{}^T \\ \mathbf{So}_{r,23}{}^T & \mathbf{s}_{r,23}{}^T \\ \mathbf{So}_{r,33}{}^T & \mathbf{s}_{r,33}{}^T \end{bmatrix} \quad (2.14)$$

which contains the screw transpose of all the reciprocal screws. And $\mathbf{J_q}$ is a diagonal matrix which has the following form:

$$\mathbf{J}_q = \mathrm{Diag}\Big[\ \hat{\$}r_{11}{}^T\hat{\$}_{11}\quad \hat{\$}r_{21}{}^T\hat{\$}_{21}\quad \hat{\$}r_{31}{}^T\hat{\$}_{31}\quad \hat{\$}r_{12}{}^T\hat{\$}_{12}\quad \hat{\$}r_{22}{}^T\hat{\$}_{22}\quad \hat{\$}r_{32}{}^T\hat{\$}_{32}\quad \hat{\$}r_{13}{}^T\hat{\$}_{13}\quad \hat{\$}r_{23}{}^T\hat{\$}_{23}\quad \hat{\$}r_{33}{}^T\hat{\$}_{33}\ \Big]$$

The analytical expressions of $\hat{\$}r_{1i}{}^T\hat{\$}_{1i}$, $\hat{\$}r_{2i}{}^T\hat{\$}_{2i}$ and $\hat{\$}r_{2i}{}^T\hat{\$}_{2i}$ are listed below. The numerator of the expressions can be used to identify the conditions of inverse singularities of the in-parallel manipulators.

$$\hat{\$}r_{1i}{}^T\hat{\$}_{1i} = -\cos\theta_{3\mathrm{i}}\mathrm{L}_{3\mathrm{i}} - \cos(\theta_{3\mathrm{i}}+\theta_{4\mathrm{i}})\,\mathrm{L}_{4\mathrm{i}}$$

$$\hat{\$}r_{2i}{}^T\hat{\$}_{2i} = \frac{4\sin\theta_{4\mathrm{i}}\ \mathrm{L}_{3\mathrm{i}}\ \mathrm{L}_{4\mathrm{i}}}{f_1(\theta_{2\mathrm{i}},\theta_{3\mathrm{i}},\theta_{4\mathrm{i}})} \quad (2.15)$$

$$\hat{\$}r_{3i}{}^T\hat{\$}_{3i} = \frac{-4\sin\theta_{4\mathrm{i}}\ \mathrm{L}_{3\mathrm{i}}\ \mathrm{L}_{4\mathrm{i}}}{f_2(\theta_{2\mathrm{i}},\theta_{3\mathrm{i}},\theta_{4\mathrm{i}})}$$

where f_1 and f_2 are non zero functions of θ_{2i}, θ_{3i} and θ_{4i}.

Since the instantaneous reference frame is coincident with the body center coordinate system $\{x_B, y_B, z_B\}$, all the reciprocal screws and joint screws are expressed with respect to the body frame.

Note that,

$$\mathbf{\$}_{\mathrm{B}} = \begin{bmatrix} {}^{\mathrm{B}}\boldsymbol{\omega}_n \\ {}^{\mathrm{B}}\mathbf{v}_0 \end{bmatrix}$$

is defined in the instantaneous reference frame as the angular velocity of the body and the linear velocity of a point (could be imaginary point) in the body that is coincident with the origin of the reference frame. However, in most operations in reality, the desired velocity state of the body is usually defined with respect to the global fixed coordinate system $\{X_0, Y_0, Z_0\}$. Therefore the transformation between the twist of the body in the instantaneous reference frame and that in global frame, which is represented as

$$\mathbf{\$}_{\mathrm{G}} = \begin{bmatrix} {}^{\mathrm{G}}\boldsymbol{\omega}_n \\ {}^{\mathrm{G}}\mathbf{v}_n \end{bmatrix}$$

must be established. These two velocity state vectors are related by the following equations:

$$\begin{aligned} {}^{\mathrm{G}}\boldsymbol{\omega}_n &= {}^{0}\mathbf{R}_{\mathrm{B}}\,{}^{\mathrm{B}}\boldsymbol{\omega}_n \\ {}^{\mathrm{G}}\mathbf{v}_n &= {}^{0}\mathbf{R}_{\mathrm{B}}\left({}^{\mathrm{B}}\mathbf{v}_0 + {}^{\mathrm{B}}\boldsymbol{\omega}_n \times {}^{\mathrm{B}}\mathbf{p}_n\right) \end{aligned} \quad (2.16)$$

where ${}^{0}\mathbf{R}_{\mathrm{B}}$ is the 3 by 3 rotational transformation matrix between the global fixed system $\{X_0, Y_0, Z_0\}$ and the body center frame $\{x_B, y_B, z_B\}$. ${}^{\mathrm{B}}\mathbf{p}_n$ denotes the position vector of the origin of body frame with respect to the instantaneous reference frame. In this particular case, since the body frame is coincident with the instantaneous reference frame, ${}^{\mathrm{B}}\mathbf{p}_n = [0 \quad 0 \quad 0]^T$.

Eq.(2.13) establishes the mapping between the velocity space of the body and the space of all nine joint rates. Actually, for a desired velocity state of the body, six actuated joints out of nine are able to drive the body to reach that state. Among all nine joints, only six of them are independent and the velocity profiles of rest three joints are dependent on the six joints due to the existence of multiple closed loops in the kinematic structure. The redundancy of the active joints provides various actuation schemes such as seven, eight or nine joints actuation. These additional joints can either passively follow a time profile or actively track it. Actuation with more than six joints can improve the performance of the robot in particular operations because the required load or speed is shared with more joint motors.

As a summary, in the inverse instantaneous kinematics, the velocity state of the body is assumed to be known. Then as long as the Jacobian matrices in Eq.(2.13) are not in singular configurations, both active and passive joint rates can be calculated. At least six joints will be chosen to track desired profiles in order to reach the desired state. Redundant actuation with additional joints can improve the performances of the robot. It is also able to eliminate the forward singularities, which is discussed in details in Section 3. In forward instantaneous kinematics, if at least six joint rates are known, then not only the velocity state of the body but also the rates of the rest joint, either active or passive, can be obtained through Eq.(2.13).

3. IDENTIFICATION AND ELIMINATION OF SINGULARITIES

Due to the existence of two types of Jacobian matrices $\mathbf{J_x}$ and $\mathbf{J}_q$ in Eq. (2.13), an in-parallel manipulator can have two types of singularities. One is inverse kinematic singularity and the other is forward kinematic singularity [9].

Inverse kinematic singularity occurs when the determinant of $\mathbf{J}_q$ is equal to zero. Under such singular configurations, the infinitesimal motion of the body of parallel manipulator along certain directions cannot be accomplished; the manipulator loses one or more d.o.f.. The inverse singularity of the in-parallel manipulators is similar to the singularity of a serial manipulator. Therefore, checking the singular configurations of individual legs is also a method to identify the inverse singularity of the whole in-parallel manipulator.

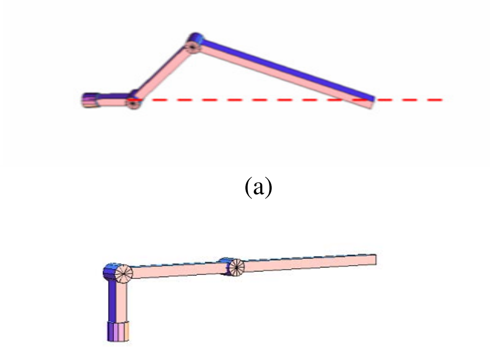
A forward kinematic singularity occurs when the determinant of $\mathbf{J}_x$ is equal to zero. Under these configurations, the manipulator gains one or more d.o.f while all actuated joint are completely locked and cannot resist wrenches in certain direction. The forward kinematic singularity of the three-legged S-R-R-R in-parallel manipulator becomes more complicated as the manipulator can implement non-redundant actuation scheme or redundant actuation scheme. Since more than six joints will be actuated in redundant actuation, $\mathbf{J}_x$ is no longer a square matrix. Identification of the singular configurations requires checking the linear dependency of each row in $\mathbf{J}_x$. Compared with conventional Jacobian matrix, screw-based Jacobian is more powerful in solving such problems. Each row of $\mathbf{J}_x$, as shown in Eq.(2.14), is the screw transpose of the reciprocal screw identified for each actuated joint. These zero-pitch screws are equivalent to the *Plücker line coordinates*, so each screw represents uniquely a line in 3D space. Using *Grassmann line geometry*, the linear dependent cases of the row vectors in $\mathbf{J}_x$ can be identified, which corresponds to the singularities of the forward kinematics.

In this section, the inverse singularities are identified at first, which shows that the inverse singular configurations of the whole in-parallel manipulator can be identified by investigating the singularities of a single leg. Since line geometry is intensively used in the identification of forward singularities, the theory of Grassmann line geometry is introduced in Section 3.2. The analytical conditions under which the forward singularities occur are identified when the three-legged S-R-R-R in-parallel manipulators are actuated in non-redundant scheme with two actuated joints per leg. Correspondingly, the elimination method based on redundant actuation is discussed. For convenience, the nomenclature n_1 - n_2 – n_3 will be used to describe the actuation scheme where n_i corresponds to the number of actuated joint angles in leg *i*. For example, the non-redundant actuation scheme with two actuated joints per leg can be described as 2-2-2. An unequally-distributed non-redundant actuation scheme can be described as 3-2-1. Since the body of the in-parallel manipulator has 6 d.o.f, at least six joints out of the total of nine joints must be actuated and each leg must have at lease one joint actuated. Redundant actuation schemes with more than six actuated joints are also possible. These cases are listed as follows: (1)3-3-3 (nine joint actuation); (2) 3-3-2(eight joint actuation); and (3) 3-3-1 and 3-2-2 (seven joint actuation).

Screw theory and line geometry were also adopted by previous researchers to analyze the forward singularities of three-legged in-parallel manipulators with different configurations such as Notash [11] and Dash [12]. Notash first proposed the elimination of forward singularities by appropriate redundant actuation scheme. The method addressed in this section can be used to identify the forward singularities of a family of three-legged in-parallel manipulators with a completely passive spherical joint at each leg [8].

3.1 INVERSE SINGULARITIES

$\mathbf{J}_q$ in Eq.(2.13) is a diagonal matrix with $\hat{\$}_{r_{ji}}{}^T\hat{\$}_{ji}$ as its diagonal elements. The conditions of inverse singularities can be derived by equating the diagonal elements to zero. According to Eq.(2.15), two types of singularities are identified. The first singularity derived from corresponds to a configuration in which the axis of the hip rotator joint is passing the foot contact point. Thus the rotational d.o.f. of the hip rotator is lost. Any input to the hip rotator cannot change the position of the body. The second singularity corresponds to a configuration in which the leg is either fully extended or fully retracted. Under this singular configuration, the body reaches the edge of its effective workspace and any motion along the direction of the shank is unreachable. Thus, the in-parallel manipulator loses one d.o.f.. These two singularities are shown in FIG.7 (a) and (b).



(a)

(b)

FIG.7. TWO CASES OF INVERSE SINGULARITY

3.2 INTRODUCTION TO GRASSMANN LINE GEOMETRY

A set of spatial lines is a *variety* if no line outside the set is dependent on the lines in the set. The varieties of spatial lines were firstly studied by H.Grassmann in 1844. The line varieties of rank 2,3,4,5 are summarized in [13]. FIG.8 provides the most representative illustrations for different types of line varieties.

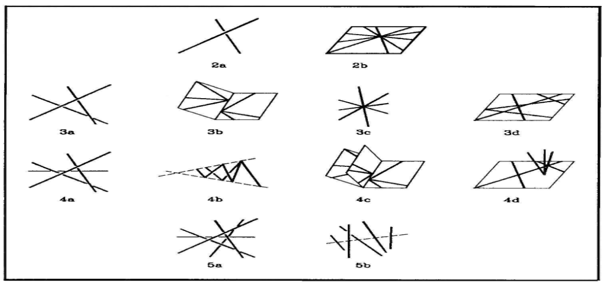


FIG.8. VARIOUS TYPES OF LNE VARIETIES (J.P.Melert,1989)

J.P. Merlet used line geometry firstly to find the singular configurations of the 6-3 Stewart Platform in [14]. The following discussions will follow the notations proposed by Merlet to describe various types of line dependency.

3.3 FORWARD SINGULARITIES AND THEIR ELIMINATION

The three-legged in-parallel manipulator, i.e the kinematic model of STriDER, is assumed to have an initial actuation scheme of 2-2-2 with each leg having two actuated joints and the reciprocal screws associated with them form a wrench plane. Both of these two reciprocal screws pass through the foot contact point, thus forming a planar pencil. The identification of reciprocal screws can refer to Section 2 and these reciprocal screws are shown in FIG.6.

The cases of the linear dependency within the three sets of planar pencils are investigated using line geometry. Each individual leg has at most three reciprocal screws and they are not coplanar except in inverse singular configurations. If certain case of linear dependency occurs, the wrench system spanned by the reciprocal screws will have an order *m* lower than 6. Under such a forward singular configuration, the body of STriDER gains extra d.o.f and cannot resist wrenches in certain directions even the actuated joints are locked. In order to eliminate the singularities, redundant actuations of 3-2-2, 3-3-2 and 3-3-3 are proposed for various cases. It is shown that all possible forward singular configurations can be eliminated with 3-3-3 actuation at worse.

In a bid to describe the geometrical relationships of the reciprocal screws and wrench planes conveniently and clearly, the terms $\mathbf{Lr}_{ni}$ and $\mathbf{Lr}_{mi}$ are used to denote the two reciprocal joint screws at leg *i*. The term $\mathbf{Lr}_{qi}$ represents the third reciprocal joint screw under redundant actuation in leg *i*. $\mathbf{X}_{mni}$ is used to represent the wrench plane generated by $\mathbf{Lr}_{ni}$ and $\mathbf{Lr}_{mi}$. The geometrical relationships identified are interpreted into vector equations to obtain the analytical conditions of forward singularities. The homogeneous coordinates of line $\mathbf{Lr}_{ni}$ and plane $\mathbf{X}_{mni}$ are represented with:

$$\hat{\$}r_{ni} = \begin{bmatrix} \mathbf{s}_{r,ni} \\ \mathbf{So}_{r,ji} \end{bmatrix} \tag{3.1}$$

and

$$\mathbf{C}_{mni} = \begin{bmatrix} \mathbf{N}_{mni} \\ n_{mni} \end{bmatrix} \tag{3.2}$$

The following figures are used to illustrate each case of forward singular configurations and the elimination scheme. $\mathbf{P}_i$ $\mathbf{P}_j$ $\mathbf{P}_k$ is the virtual base triangle of the manipulator. The effective redundant reciprocal screw is represented with diamond arrow while the ineffective reciprocal screw is represented with oval arrow.

Case 1 – Collinear lines:

Two zero-pitch screws are dependent if they lie on the same line. Such two screws can only constraint one translational d.o.f. along the axes of reciprocal screws. Case 1 occurs when one reciprocal screw of leg *i*, is collinear with a reciprocal of leg *j*. For the three-legged S-R-R-R in-parallel manipulator, this case is possible only if the reciprocal screws are collinear with P_iP_j. This case is shown in FIG.9. The condition under which this case occurs is that $\mathbf{Lr}_{ni}$ passes $\mathbf{P}_j$ and $\mathbf{Lr}_{nj}$ passes $\mathbf{P}_i$. The vector equations are written as follows:

$$\begin{aligned} \mathbf{P}_j \times \mathbf{s}_{r,ni} &= \mathbf{So}_{r,ni} \\ \mathbf{P}_i \times \mathbf{s}_{r,nj} &= \mathbf{So}_{r,nj} \end{aligned} \tag{3.3}$$

Redundant actuation at leg *j* is able to eliminate this singularity. The additional reciprocal screw is represented with a diamond arrow in FIG.9. Note that, if Case 1 occurs doubly or triply, then 3-3-2 actuation and 3-3-3 actuation are used to eliminate the singularities.

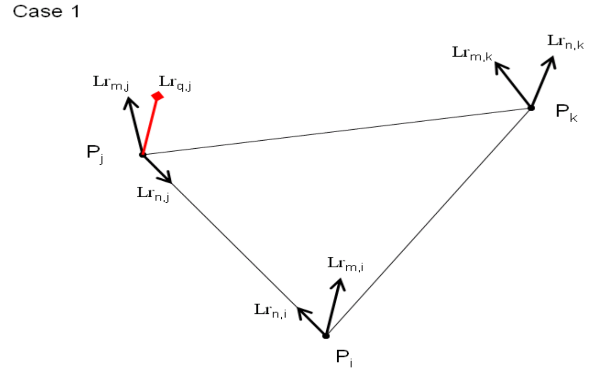


FIG.9. CASE 1: COLLINEAR LINES

Case 2a – Two skew lines:

Two skew zero-pitch screws can not generate a third screw that is a linear combination of the two screws. So this case cannot happen.

Case 2b – Coplanar concurrent lines:

Two intersecting lines define a plane. If a third line lies in the plane and passes through the concurrent point of the two screws, Case 2b will happen. The order of the flat pencil defined by the three lines is 2. This case is shown in FIG.10.

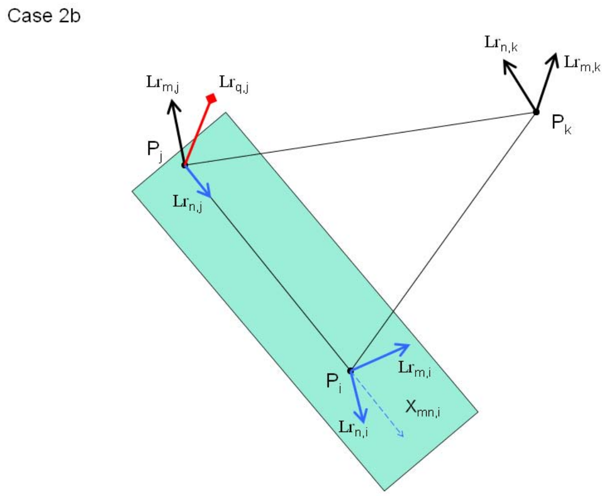


FIG.10. CASE 2b: COPLANAR CONCURRENT LINES

The condition under which Case 2b occurs is that $\mathbf{Lr}_{nj}$ has the same direction as $\mathbf{P}_j\mathbf{P}_i$ and $\mathbf{P}_j$ lies on the plane $\mathbf{X}_{mni}$. The vector equations are as follows:

$$\begin{aligned} &\mathbf{s}_{r,nj} \times \mathbf{P}_j\mathbf{P}_i = 0 \\ &\mathbf{P}_j \cdot \mathbf{N}_{mni} = n_{mni} \end{aligned} \tag{3.4}$$

The redundant actuation with 3-2-2 can eliminate this singularity. The effective additional reciprocal screw is represented with diamond arrows in FIG.10. If Case 2b occurs doubly, then 3-3-2 actuation with two additional active joints is able to eliminate it

Case 3a – Regulus:

This case doesn't exist for the three-legged S-R-R-R in-parallel manipulators, because the maximum number of skew lines in the reciprocal screw system of the manipulator is only three.

Case 3b – Union of two planar pencils:

This case will occur when the common line of the plane defined by two reciprocal screws in leg i and j coincides with the side of the base triangle $\mathbf{P}_i\mathbf{P}_j$. The screw system has an order of 3. As shown in FIG.11, four lines $\mathbf{Lr}_{nj}$,$\mathbf{Lr}_{mj}$, $\mathbf{Lr}_{ni}$ and $\mathbf{Lr}_{mi}$ form a union of two flat pencils with the common line of $\mathbf{P}_i\mathbf{P}_j$. The condition of this case is that $\mathbf{P}_\mathrm{j}$ lies in $\mathbf{X}_{mni}$ and $\mathbf{P}_\mathrm{i}$ lies in $\mathbf{X}_{mnj}$. The vector equations are written as:

$$\begin{aligned} &\mathbf{P}_j \cdot \mathbf{N}_{mni} = n_{mni} \\ &\mathbf{P}_i \cdot \mathbf{N}_{mnj} = n_{mnj} \end{aligned} \tag{3.5}$$

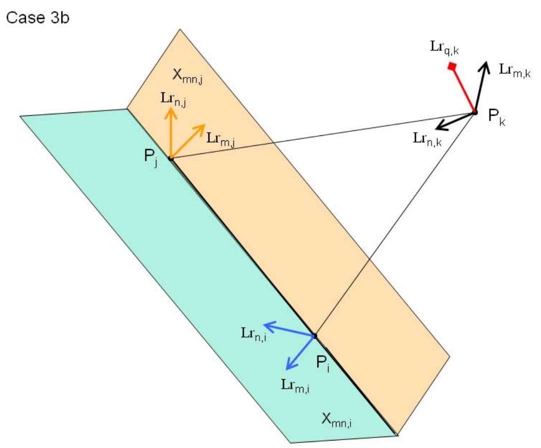


FIG.11. CASE 3b:UNION OF TWO PLANAR PENCILS

Case 3c – A bundle of lines

This case is shown in FIG.12, both $\mathbf{Lr}_{nj}$ and $\mathbf{Lr}_{mk}$ pass $\mathbf{P}_i$ and then four lines intersect at one point. The condition of this case is similar to Case1. Note that, the redundant actuation of leg i, denoted with an oval arrow, is not able to eliminate the singularity. The effective additional reciprocal screw is again represented with a diamond arrow.

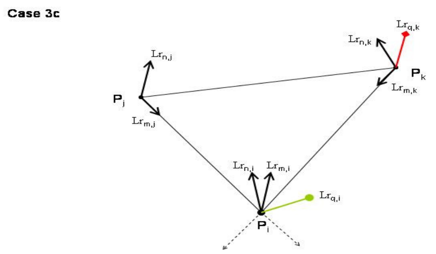


FIG.12. CASE 3c: A BUNDLE OF LINES

Case 3d - Coplanar non-concurrent lines

This case arises when at least four non-concurrent lines are lying on the same plane. Assume that the virtual base plane of the robot is the plane where four reciprocal screws lie, then this case is demonstrated in FIG.13. The criteria that a line passing $\mathbf{P}_\mathrm{i}$ or $\mathbf{P}_\mathrm{j}$ or $\mathbf{P}_\mathrm{k}$ is that the direction vector of the line is perpendicular to the normal vector of the virtual base plane.

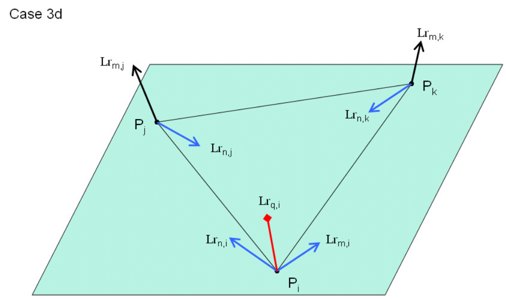


FIG.13. CASE 3d: COPLANAR NON-CURRENT LINES

Case 4a - Four independent skew lines:

Similar to Case 3a, it is impossible to have five skew lines in the reciprocal screw system of this manipulator.

Case 4b - Lines concurrent with two skew lines:

In order to investigate the condition under which this case occurs, the potential lines that intersect five reciprocal screws must be identified first. Once two candidate lines are chose, then by taking the reciprocal product of the two candidates, whether or not the two lines are skew can be determined. FIG.14 demonstrates such an example.

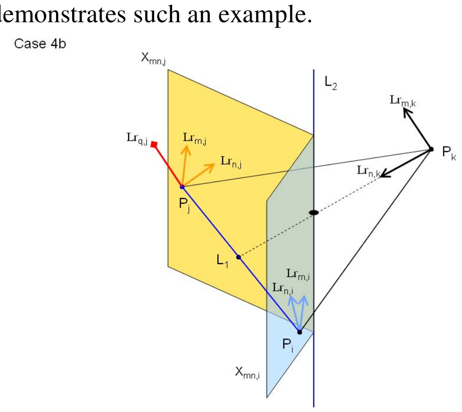


FIG.14. CASE 4b: LINES CONCURRENT WITH TWO SKEW LINES

As shown in FIG.14, the first candidate is line $\mathbf{L}_1$, i.e. $\mathbf{P}_i\mathbf{P}_j$. If $\mathbf{Lr}_{nk}$ lies on plane $\mathbf{P}_i\mathbf{P}_j\mathbf{P}_k$, then $\mathbf{L}_1$ intersects five lines simultaneously. The second candidate is line $\mathbf{L}_2$, which is the common line of $\mathbf{X}_{mnj}$ and $\mathbf{X}_{mni}$. If $\mathbf{L}_2$ also intersects $\mathbf{Lr}_{nk}$, then it will intersect the same five lines as $\mathbf{L}_1$. If $\mathbf{L}_1$ and $\mathbf{L}_2$ are two skew lines, then these five lines form a linear congruence with order 4. The reciprocal screw $\mathbf{Lr}_{qj}$ represented with diamond arrow, which is not coplanar with $\mathbf{X}_{mnj}$, can effectively elimination this singularity.

Case 4c - One parameter family of flat pencils:

This case occurs when the centers of the three planar pencils lie on the same line and this line is also the common line of the three wrench planes. This case is impossible for the manipulator discussed in this paper because the virtual base plane $\mathbf{P}_i\mathbf{P}_j\mathbf{P}_k$ is assumed to have a triangle shape.

Case 4d - Lines on a plane or passing through one point of the plane:

This case exists within at least five reciprocal screws where a minimum of two screws must be coplanar and the remaining screws, which must be larger or equal to one, intersect the plane defined at one point. The two-system of unconstrained d.o.f. can be described as combinations of twists about two orthogonal axes lying on the plane and having directions perpendicular to the normal to the plane passing the intersection point. An example of Case 4d is shown in FIG.15. Additional reciprocal screw $\mathbf{Lr}_{qk}$ at $\mathbf{P}_k$, represented with a diamond arrow, is able to eliminate this singularity.

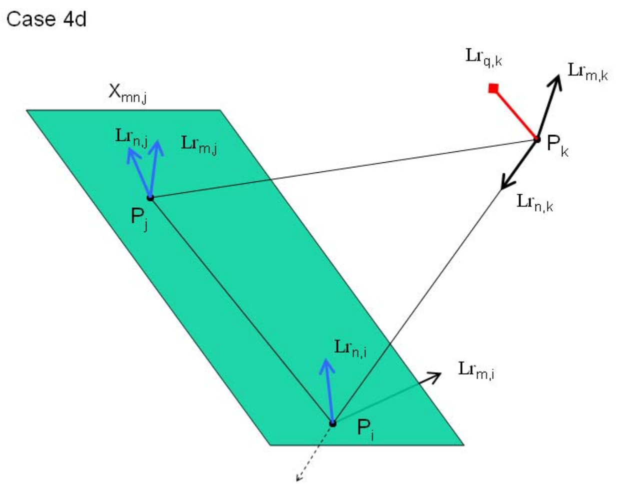


FIG.15. CASE 4d: LINES ON A PLANE AND PASSING A POINT

Case 5a - General complex:

If one screw can be expressed as a linear combination of the other five independent screws, the variety of the six screws is called a *complex,* which has an order of 5. The coplanar lines of a complex meet at a common point. This property can be utilized to identify whether six screws belong to a complex. The example of this case is shown in FIG.16. However, as for the SRRR in-parallel manipulator, this case becomes the same as Case 5b.

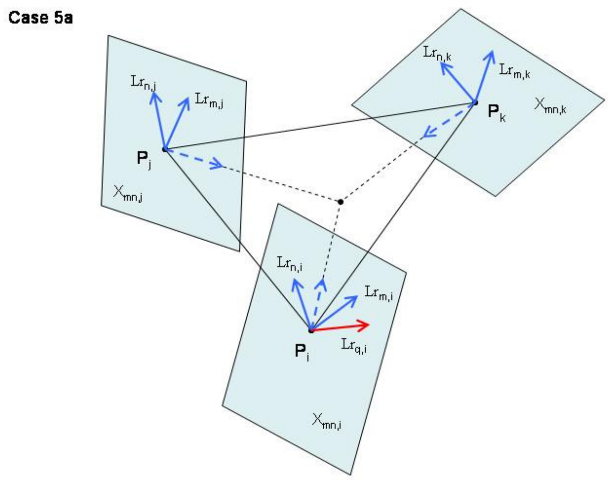


FIG.16.CASE 5a: GENERAL COMPLEX

Case 5b – Special complex:

Case 5b happens when six reciprocal screws intersect one line. The unconstrained d.o.f is any rotation along the common line. One example is shown in FIG..17, the redundant actuation of any leg is able to eliminate the singularity.

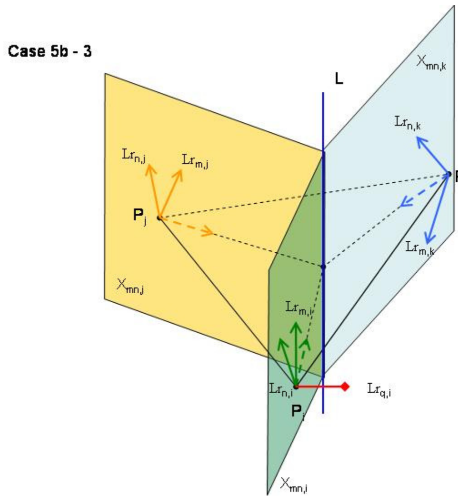


FIG.17.CASE 5b: SPECIAL COMPLEX

As a summary, line geometry is able to identify all possible forward singular configurations. The redundant actuation can eliminate the forward singularities by introducing appropriate additional reciprocal screws, thus increasing the order of the forward Jacobian matrix. The actuation scheme of 3-3-3 is able to eliminate all possible forward singularities of the three-legged in-parallel manipulator.

4. EXAMPLES OF FORWARD SINGULARITIES

This section mainly presents the representative examples of the forward singular cases discussed in Section 3.

FIG.18 presents an example of Case 1. In this case, the flexure joint and knee joint in each leg are chosen as the active joints. The wrench plane of leg 1 and leg 2 are coincident with the plane of the body. The associated reciprocal screws to the flexure joints in leg 1 and 2 are collinear. These six reciprocal screws belong to a line variety with the order of 5. Assume the reciprocal screws in leg 3 intersect the body plane at two points. Then the unconstrained d.o.f. is a rotation along the line in the body plane that passes the two points. Redundant actuation at any leg can eliminate this singular configuration.

FIG.19 presents an example of Case 2b. The actuated joints are the knee joints and flexure joints. Again, the wrench plane of leg 1 and 2 are coincident with the body plane. The reciprocal screw associated with the flexure joint of leg 1 passed the foot contact point of leg 2. There reciprocal screws make up a planar pencil with order 2. The unconstrained d.o.f. is the same as the case shown in FIG.18. The elimination method is similar.

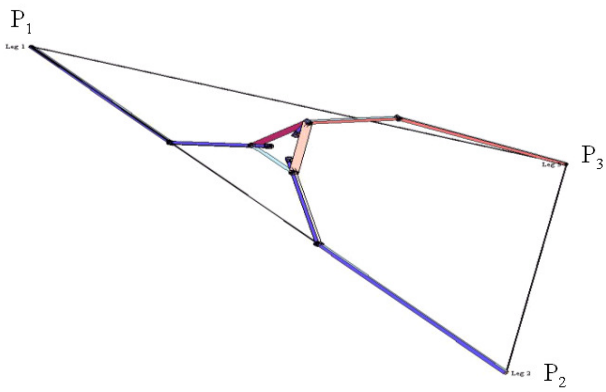


FIG.18.EXAMPLE OF CASE 1

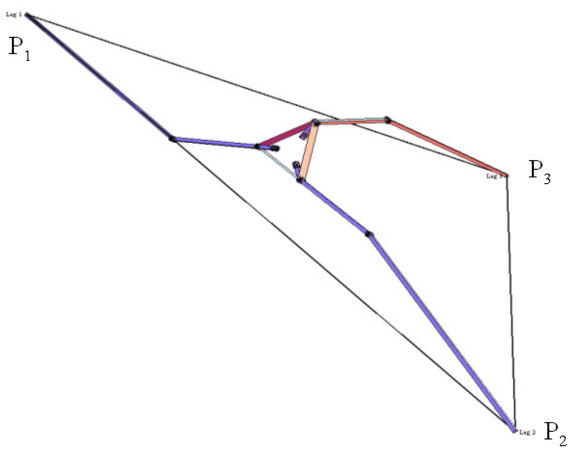


FIG.19. EXMAPLE OF CASE 2b

The example of Case 3d singularity is displayed in FIG.20. Four non-concurrent reciprocal screws lie on the body plane. The unconstrained 1 d.o.f. is the same as the previous two examples and the elimination method is similar.

FIG.21 shows the example of Case 4d. Assume that STriDER is actuated with 3-2-1 scheme. The wrench plane of leg 2 contains the foot contact point of leg 1. If all three joints at leg 1 are actuated. Then the five reciprocal screws constitute a line variety with order 4. One reciprocal screw at leg 3 will intersect the body plane at one point. A straight line that connects the point with the foot contact point at leg 1 will intersect all six reciprocal screws. The unconstrained d.o.f is a rotational motion along that line. Redundant actuation at leg 3 can eliminate this singularity.

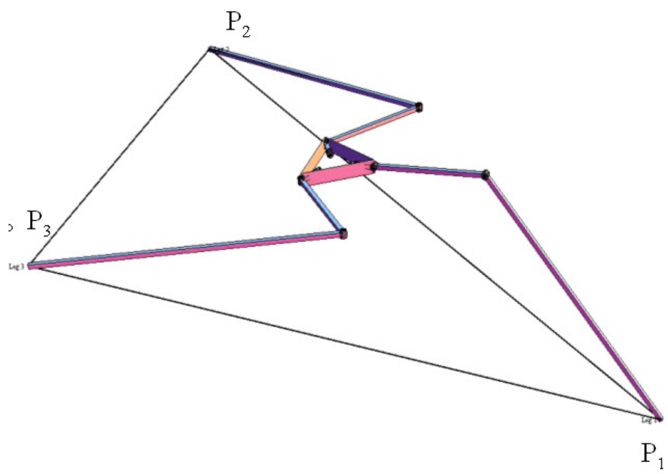


FIG.20.EXAMPLE OF CASE 3d

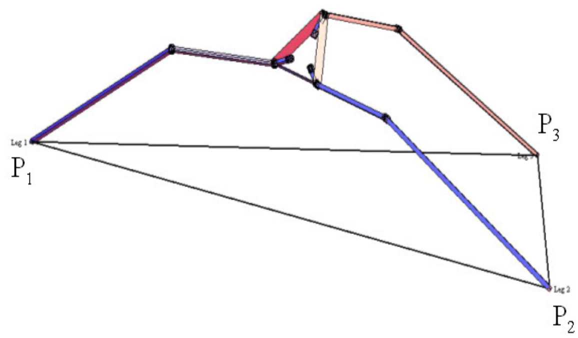


FIG.21.EXAMPLE OF CASE 4d

FIG.22 shows the example of Case 5b. Although in this figure, STriDER stands on the ground like a stable tripod, however, the robot is actually in a forward singular configuration if in each leg only the flexure joint and knee joint are actuated. The three wrench planes have a common line which passes the center of the body. All six reciprocal screws intersect this common line. The screws parallel to this line is considered as intersecting at infinity. Therefore, the robot has one unconstrained rotational d.o.f along the common line. Actuation of any rotator joint will eliminate this singularity.

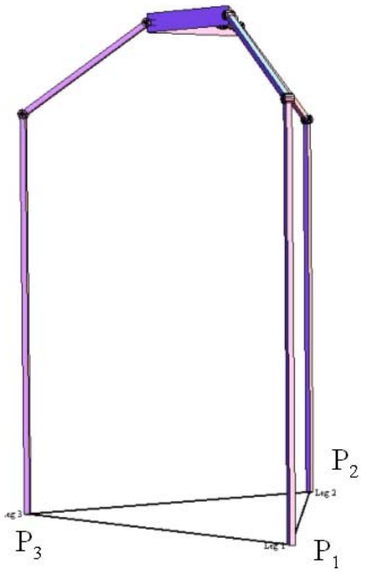


FIG.22.EXAMPLE OF CASE 5b

5. CONCLUSION AND FUTURE RESEARCH

In this work, a novel mobile robot STriDER is modeled as a three-legged in-parallel manipulator when all three feet are in contact with the ground without slipping. Screw theory and line geometry are adopted in the study of the instantaneous kinematics and singularity analysis of this manipulator. The based Jacobian matrices are developed. The velocity of the body can be controlled with more than six actuated joints. With the assistance of line geometry, the forward singular configurations are identified. Actuation with more than six joints can effectively eliminate those singularities, thus allowing the manipulator to resist the disturbance forces and moments.

Future research on the singularity of STriDER will focus on the singularity locus and the investigation of a checking algorithm that can efficiently monitor the order of the reciprocal screw system in real time and adopt appropriate redundant actuation to eliminate forward singularities. Since the feet of this robot are not firmly fixed to the ground, it is necessary to study the statics of the robot and investigate the force reactions between the feet and the ground.

ACKNOWLEDGMENTS

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