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# CONTACT STRESS REDUCTION MECHANISMS FOR THE CAM-BASED INFINITELY VARIABLE TRANSMISSION

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#### ABSTRACT

The Cam-based Infinitely Variable Transmission (IVT) is a new type of ratcheting IVT based on a three dimensional cam and follower system which provides unique characteristics such as generating specific functional speed ratio outputs including dwells, for a constant velocity input. This paper presents several mechanisms and design approaches used to improve the torque and speed capacity of this unique transmission. A compact, lightweight, and capable differential mechanism based on a cord and pulley system is developed to double the number of followers in contact with the cam at any time, thereby reducing the contact stress between the followers and the cam surface considerably. A kinematic model governing the motion of this differential is developed and a few experimental results from the prototype are presented, showing an overall increase in performance including a smooth output, a wide gear range, and the ability to shift under load. Plans for future improvements to the design, including an inverted external cam mechanism, is also presented along with the expected performance gains.

#### **1 INTRODUCTION**

The Cam-based IVT is a unique transmission with many capabilities beyond that of other current ratcheting drives and IVT technologies such as the torroidal and belt driven type. Much like the torroidial drives, in which most if not all of the driving torque passes through a small contact region [1], the entire force reaction to the driving torque of the Cam-based IVT is transmitted through the contact area between the rollers and cam. Therefore, this small region exhibits a very high Hertzian contact stress which is proportional to the input torque and the gear ratio. For a given transmission size, it was found that this contact stress is the limiting factor of the torque capacity of the

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transmission. While other highly stressed components can be resized to accommodate high loads, such as the sprag clutches, input shaft, and planetary gear system, the cam and roller system has a much greater effect on the overall transmission size than the other components. So for a given input torque, the overall size of the transmission was determined such that this contact stress was within the limits of both the cam and roller material.

The research and mechanical design of the transmission was focused on reducing this contact stress through either the reduction of the contact force on the roller, or minimizing the stress resulting from this force. To reduce the contact force, increasing the number of followers, number of rollers, and/or changing the transmission parameters were studied. Aside from minimizing the contact force, changing the diameters of the contacting bodies, reducing the modulus of elasticity of the materials involved, or changing the geometry of the contact region were all suitable means of minimizing the stress for a given contact force.

Our end approach was two pronged, to at least double the number of rollers under load at any time, and also to modify the overall topology of the mechanism to incorporate an inverted and external cam surface which surrounded the rest of the mechanism. Utilizing a unique cable differential, which splits the input torque evenly between two followers, allowed for two active followers and therefore loaded rollers. Inverting the cam provided a larger radius of curvature, and a more complimentary surface for the roller to follow. Both methods and their characteristics are presented here.

# 1.1 Cam-based IVT Operation

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The operation of the transmission has been presented before, but is summarized here for convenience [2]. At the center of the transmission is a three dimensional cammoid around which are a number of followers, rotatably mounted to a carrier plate as seen in Figure 1. The carrier rotates around the central axis of the cam, causing the followers to orbit around the cam. Each of these followers is fixed to a planet gear or similar planet pulley. There exist a number of sun gears, each meshed to at least one of the planet gears. It is this connection, between two planet gears and the sun gear, in which the cable based differential, is used. All the sun gears are connected to the output shaft through an indexing clutch.



Figure 1. CAD model of the Cam-based IVT.

In its simplest inversion, a rotational input to the carrier causes followers to move around the cam. The followers will oscillate as they move up and down the lobes of the cam, causing the planet gears and the connected sun gears to oscillate as well. Here the operation is similar to a planetary gearset, in which the rotation of the sun gear depends both upon the carrier and the planet gear rotations. In this transmission, the oscillations of the sun gear due to the planet gear are superimposed upon the forward rotation of the sun gear due to the carrier. As the individual sun gears oscillate, they will periodically engage and disengage the output shaft through the indexing clutches. During the engagement period, the followers are said to be in the active region of the cam.

By stacking an infinite series of profiles along the length of the cam to make a three dimensional cammoid, an infinite number of transmission ratios can be selected. By varying the position of the cam followers in relation to the cam, the particular profile they follow can be changed. This affects the magnitude of the follower's oscillations and therefore the output of the transmission. In its current implementation, all the followers are connected to a single ring which rolls through a pair of guides. These guides are actuated by a pair of lead srews, which can move the ring, and therefore followers across the cam, as in Figure 2.

There are several different inversions to this mechanism in which the input and output can be varied between the cam, carrier, and sun gear. The inversion studied here utilizes the sun as the input and the carrier as the output.



Figure 2. Shifter ring and ring guides on the prototype.

#### **2 ISSUE OF CONTACT STRESS**

In the Cam-based IVT there is a direct connection between the input gear and the followers, such that the entire reaction force to the driving torque is transmitted through the contact region between the roller and the cam. Therefore, this small region exhibits a very high Hertzian contact stress specifically during the active region of the cam, when the most significant loading of the followers occurs.

# 2.1 Contact Stress Model

A method for finding the contact force between the roller and cam has been presented [2], and this method was used in these calculations for the active region considering only static loads. All design approaches and mechanisms were evaluated using these calculations. At the time of writing, the transmission specifications called for a maximum input torque capacity of 90 [ft-lbs] and an input rpm of 150 [rpm]. To correctly model the contact stress, the radius of curvature of the pitch curve and the cam surface itself were estimated as if the system was a planar cam mechanism [5]. Therefore, the effect of the draft angle of the cam was not modeled as part of the cam surface radius, but was taken into account within the contact force model. The effect of this draft angle on the radii calculation is small and so this is a safe assumption. In the calculation of the Hertzian contact stress, the roller and cam are assumed to be spherical. Therefore, the contact stress will be slightly over-estimated if one considers the cam surface is actually a ruled surface and could be more closely modeled as a cylinder.

#### 2.2 Methods Contact Stress Reduction

A systematic approach was taken to minimizing this contact stress by beginning with a look at the Hertzian contact stress equations. Considering the geometry of the contacting bodies first, the effect of the diameters on the contacting area is given by Equation 1:

$$eff = \frac{1}{d_1} + \frac{1}{d_2} \tag{1}$$

where  $d_1$  and  $d_2$  are the diameters of the two contacting bodies [3]. Therefore, changing the diameters such that Equation 1 is minimized will result in the largest contact area and therefore smallest stress. Either increasing the diameter or using a negative diameter, equivalent to utilizing the inner surface of a hollow sphere, will suffice. Such was the goal of utilizing the inner surface of an inverted and external cam. This approach also reduces the contact stress through an improved topology which allows the highly stressed components to be made larger while transmission size remains the same.

Changing the geometry such that cylindrical contact is present as opposed to spherical will also introduce a significant reduction in the contact stress. This is a result of a larger contact area between bodies for a given deformation. Such an approach has been tried in a similar ratcheting IVT also based on a cam drive which can be seen in Figure 3.



Figure 3. Model of Naude's cam driven IVT with cylindrical rollers [4].

While the use of cylindrical rollers in conjunction with a three dimensional cam can reduce the contact stress, the complexity is increased due to an additional degree of freedom that is required to allow the rollers to rock back and forth as they follow the draft of the cam. More significantly, sliding between the roller and the cam is also more prevalent as the ends of the rollers follow different cam profiles. Therefore this approach was not followed, but rather the design, materials, and other mechanisms within the transmission were emphasized.

When using aluminum or plastic as the cam and roller material, this contact region exhibits the highest stress within the transmission. If the design were to remain the same, higher performance materials could be substituted for the cam to achieve a greater torque capacity. In particular, titanium was investigated for its low modulus and high strength properties. Because of its high cost and the fact that it is difficult to machine, this approach was abandoned. Although, this could be implemented after other mechanism and design improvements to increase performance.

Minimizing the contact force is more difficult as it depends on transmission parameters which are more closely tied to the design criteria such as the overall transmission ratio. To achieve this reduction then, distribution of the contact force was emphasized. Both a dual roller follower, and a dual active follower system were studied as means of accomplishing this and require unique mechanisms for utilization such as the cable differential presented here.

#### **3 DUAL ACTIVE FOLLOWER SYSTEM**

Increasing the number of active rollers in contact with the cam allows the input load to be distributed across more rollers and therefore decrease the contact stress. This can be accomplished in two different ways. Firstly, the number of followers can be increased, such that two followers, and hence rollers, will be active at any time, such as the model in Figure 1. The second option is to add an additional roller to each follower, which can be seen in Figure 4 next to the current follower design. In the latter case, each pair of rollers is mounted on a "truck" such that they are loaded equally. The former was researched first because the solution could be more readily implemented. In addition, at the time of writing, no literature was found regarding the method required to determine a cam surface for a dual roller follower. In the future, both solutions could be implemented simultaneously for further improvement and a method of finding the correct cam surface will be investigated.



Figure 4. The current follower system on the left and the dual roller follower concept.

#### 3.1 Benefits

Aside from the reduction in contact stress the dual follower system allows, there are some additional benefits. Considering the static case, the net reaction on both the cam and the carrier will be reduced to a moment only, thereby reducing the load on any supports. Had there been only one active follower, or two active followers located adjacent to each other as opposed to across, the normal forces from the followers on the cam would not cancel, and the resulting force can be very large, in our case almost 1000 [lbs].

In addition, there are several advantages when looking at the dynamics of the mechanism. By placing the followers across from one another, the dynamic imbalances that could be present as the followers rotate with respect to the carrier are minimized or completely eliminated. This particular follower arrangement also produces an axial symmetrical cam, one which is dynamically balanced, which is ideal for inversions which require a rotating cam.

It was predicted that moving from one to two active followers would have a significant impact on the contact stress. It was necessary to test the hypothesis because although the followers would see a reduced load, the higher acceleration required of the shorter acting profile would decrease the cam surface curvature. The system was modeled in Matlab® with a carrier radius of 2.5 [in], follower length of 1.5 [in], and a roller radius of .5 [in]. A transmission ratio of 3 was achieved with a follower velocity of 0.30 [rad/rad] and a planet to sun ratio of 2.2. With four total followers, one being active at a time, the maximum contact stress was 49.4 [ksi] with a maximum shear stress of 14.9 [ksi]. Using the cable differential with the same follower velocity, the stress is reduced by 41% to 29 [ksi] and 7.6 [ksi] respectively.

#### **3.2 Differential System Requirements**

Ideally, two followers could be engaged at any time by placing a second planet gear on the same sun gear as the opposite follower. The operation of this transmission though precludes this solution for two reasons. The first is that manufacturing tolerances in the cam, carrier, or follower may allow one follower to disengage from the cam, thereby overloading the second follower. This would occur for instance if the cam surface was undersized on one side of the cam. Secondly, because of the unique shifting mechanism of this design, which allows for shifting while stopped or under load, the followers may not necessarily be on the same cam profile. Thus the one on the more aggressive profile will become the sole active follower leading to a similar overloading situation as above. It is clear that a differential type mechanism is needed to split the load equally between the two while allowing them to move relative to each other. For this application, a unique planar cable differential was developed.

#### 3.3 Prior Art

The constraints for the mechanism were a narrow width, light weight, ease of construction, and high torque capacity. Because of the nature of the transmission, the relative motion between the two inputs, the planet pulleys, would remain small. The maximum swing of the follower in this transmission is around 45 degrees; therefore, a continuous motion device was unnecessary. While a conventional differential would work and was investigated, these are generally more expensive (due to manufacturing), heavier, and complicated then a cable based system due to the nature of force transmission through conventional gear teeth. In addition, a cable based system is less limited by the selection of stock gears.

The use of cable based differential and power transmission systems is commonly seen in robotic applications, where low

backlash, high accuracy, and low weight are of primary concern [6]. These make the mechanisms ideal for transmission of power over a distance with few drawbacks. In both the Schillebeeckx and Jacobsen mechanisms, these differentials drive an output which is at a right angle to the inputs, as can be seen in Figure 5 [6,7]. In these cases, with a perpendicular drive, the difficulty becomes routing the cable through the pulleys while keeping the cable from becoming wedged between the pulleys themselves. In addition, because of the planar topology of the planetary gearset within the transmission, these existing cable differentials are not suitable.



Figure 5. Cable driven differential drive mechanism. (reprinted from Schillebeeckx[7])

# 3.4 Cable Differential Design

The particular solution was chosen because of its simplicity, light weight, and high torque transmissibility. The design consists of a rope or cable wrapped around a pinion pulley, where the tension, T in both the ropes is equal. Referring to Figure 6, the force on the upper pinion pulley is then 2T. If the free ends of the cable are wrapped around two other input pulleys, such that a torque can be transmitted through them, the reaction force for the pinion pulley on its mounts is directly related to the input torques. Attaching the pinion pulley to a central sun pulley will result in a differentially geared pulley system as seen to the right in Figure 6. The center pulley then becomes analogous to the carrier of a conventional automotive differential while the small pulley attached to it is similar to the pinion gears. The output force then becomes an output torque, which is directly related to the input torques on the planet pulleys.



**Figure 6.** The cable differential system used in the CVT, right, is an adaptation of a conventional pulley system.

One of the primary concerns when incorporating the cable differential into the Cam-based IVT was the routing path of the cable around the pulleys. The path must allow for a sufficient range of motion of all three pulleys, be of narrow width, and must minimize additional components. Two such cable routing examples which were investigated can be seen in Figure 7, the difference being that one routes the cable through the inside of the sun pulley while the other is routed entirely around the sun pulley. In the most recent IVT prototype, the interior region of the sun pulley was occupied with a sprag clutch, and therefore the cable was routed around the sun gear, running back against itself for a portion of the circumference. The former routing option could be implemented if the width of the mechanism was a lower priority and the cable on cable interaction was a concern. They are otherwise identical in operation.



Figure 7. Two optional cable routes for the cable differential, one through the middle of the sun pulley, one which is routed around the sun.

#### 3.5 Kinematic Equation

The equation of motion of this system is derived by assuming the chord is inextensible. Using this assumption, the total length of the chord at any pulley position is:

$$L = L_0 + 2r_3\theta_3 + r_{PA}\theta_{PA} + r_{PB}\theta_{PB}$$
(2)

where  $\theta_{PA}$  and  $\theta_{PB}$  are the amounts of cable wrap in radians of the planet pulleys,  $\theta_3$  the wrap around the sun pulley, *r* is the respective pulley's radius,  $L_0$  is the sum of the lengths of cable not wrapped around a pulley, and *L* is the total length of the chord. Taking the derivative of Equation 2 yields:

$$r_{PA}\omega_{PA} + r_{PB}\omega_{PB} = -2r_3\omega_3. \tag{3}$$

 $L_o$  is constant because the cable is designed to remain tangent to the pulleys at all times and this tangency point will remain at the same location with respect to the carrier. Also, the pulley rotation axes do not move in relation to one another. A similar relationship can be derived by looking at the energy flow through the system. If power is give by torque times velocity, conservation of energy states that:

$$Tr_{PA}\omega_{PA} + Tr_{PB}\omega_{PB} + 2Tr_3\omega_3 = 0 \tag{4}$$

where T is the tension in the cable and can be divided out. The result will be the same as Equation 3. In practice, the differential is treated as a standard gearset because the followers are assumed to be on the same profile for most of the transmissions operation and this simplifies the calculations.

Studying Equation 4, it can be shown that a second advantage of the cable differential over a geared version is that the torque split can be varied by changing the relative pulley sizes. A comparable feature on a bevel gear differential would require gears cut at non-conventional angles. Also, with this cable differential, it is possible to vary the pulley size ratios throughout the travel of the mechanism, much like noncircular gears. The manufacturing of such pulleys though is considerably easier then that of non-circular gears. If such a design were to be used, the motion of the transmission would be the same as Equation 3, except  $r_p$  would be a function of the follower position. This characteristic is of particular interest because it would allow the pressure angle of the Cambased IVT follower to be tailored to specific points of the follower motion for improved force transmission.

#### 3.6 Mechanical Design

The experimental prototype seen in Figure 2 was constructed from aluminum and plastic to verify the operation of the differential under both normal operation and during shifting. A detail shot of the differential mechanism can be seen in Figure 8.



Figure 8. Transmission prototype with cable differential.

A dual active follower system requires a different cam profile then a single follower design. There are now two identical active profiles per revolution, as seen in Figure 9 for a follower velocity of 0.3 [rad/rad]. This requires a much higher acceleration of the followers if no changes are made to the follower profile function. This also as the effect of reducing the angular range of travel of the follower, which is beneficial when using the cable based differential.



Figure 9. Comparison of the single follower cam profile to that of the dual follower design.

The performance of the differential system was found to be more than adequate. While the input load applied remained small due to the limits of the prototype, the performance of the differential was confirmed. At a constant gear ratio, the output of the transmission appeared smooth, and with a planet to sun ratio of 1.8, an overall transmission ratios between 1 and 2.5 was achieved. As can be seen in Figure 10, the flexibility of the shifter ring is confirmed, allowing the followers to be approximately half an inch apart on the cam surface. Such a situation occurs when the IVT is shifted while either under load or while stationary. Upon leaving the active region of the cam, the followers begin moving across the cam and finally reach the new cam profile as the ring moves through the shifter guides. As the transmission operates with followers at different positions, the differential is able to maintain contact between the rollers and cam. This is evidenced by the wear present on the aluminum sun pulley from the cable sliding to adapt to the different follower velocities. This amount of wear though is not significant and it is believed this will not affect the durability of the mechanism.



Figure 10. Maximum offset of followers during normal operation.

We have also found that the maximum torque carrying capacity of this differential is primarily limited by the type of cord being used, which in general, is higher then a geared system of similar weight. This is the case because the system is non-continuous, and therefore all the connections between the cord and pulleys are hard, as opposed to friction based connections of an endless belt mechanism. In our case we are using a Kevlar® line with 130[lbs] working limit. Future versions though will call for Spectra® material based cable with a much higher strength. The polymer based cables are chosen for because they can be wrapped around smaller radii then a comparable metallic cable.

# **4 EXTERNAL INVERTED CAM**

While future versions will all incorporate the cable differential and dual active followers for their many improvements, development is continuing towards incorporating an external cam which dramatically reduces the contact stress for a given contact force. A partial model is presented in Figure 11 for reference.



Figure 11. Partial CAD model of transmission including the external cam, followers, and carriers.

# 4.1 Benefits

In addition to the cam being larger as compared to an internal cam such as in Figure 1, the most dramatic improvement in contact stress results from a now negative radius of curvature as the spherical roller is now on the inner as opposed to the outer surface of the cam. Thus Equation 1 is reduced, and the resulting contact area between the roller and cam is increased with less deformation of the roller. For example, using a carrier radius of 2.5 [in], a follower length of 1.5 [in], and a roller of .5 [in], the older, internal cam produces a maximum pressure of 41 [ksi] on the cam surface with a maximum shearing stress of 12 [ksi]. Using the same dimensions and material properties, which is an unlikely situation as will be described later, the maximum pressure and shear stresses are reduced 37% to 25 [ksi] and 7.6 [ksi] respectively.

The use of an external cam offers several other improvements in terms of packaging. With an internal cam, there was a large region of space inside the cam which could not be utilized. With the external cam, this space is moved to the outside of the transmission, freeing up a large amount of space on the inside. Being as such, the shifting mechanism can be moved to the inside, and the size of the more highly stressed components can be increased while the overall transmission size remains the same. For example, with the external cam, the carrier radius can be increased from 2.5 to 2.7 [in], and the roller from 0.5 to 0.7 [in], further reducing the contact pressure to 19.7 [ksi] and the shear to 5.9 [ksi].

Although this transmission has a uniform output velocity given a constant input, a significant problem with the early prototypes was the non-uniform torque output. This was the result of the heavy follower return springs storing and releasing energy as they held the followers to the cam. Because of the initial internal cam design, large springs were needed to hold the rather massive followers to the cam at high velocities. In addition, the higher follower acceleration required with the dual active followers exacerbated the problem. Indeed, it was experimentally determined that even heavier springs were needed then were built into the design. As such, the external cam will reduce or eliminate the need for large springs because the inertia of the followers will naturally force them onto the cam surface. Therefore lighter springs can be used which are required to only maintain tension in the cables of the cable differential. The torque fluctuations can then be almost entirely eliminated in future prototypes.

#### 4.2 Incorporation of the External Cam

The cam profile generation algorithm is almost identical to the internal cam; the same follower envelope equations were used with only a sign change as described in Tsay [8]. To maintain the correct follower rotation direction and force, the rollers now lead the follower, as opposed to trailing. Also, to eliminate the interference between the follower and the cam, the follower has been redesigned from that seen in Figure 4 to that in Figure 12. The main body of the follow has been eliminated, and the roller now sits on the primary cross shaft. In addition to reducing the part count and the follower weight, this allows the carrier radius to be increased beyond the minimum inner radius of the cam, as can be seen in Figure 11. With this cross shaft going through the middle of the roller, the minimum roller radius is limited such that the shaft does not interfere with the cam surface. Although a larger roller radius is generally desired to reduce contact stress, this minimum limit becomes an issue when the roller size must be reduced to prevent undercutting of the cam.



Figure 12. CAD model of the follower and roller assembly to be used with the external inverted cam design.

The shifter ring will remain similar to prior designs, but will most likely be made more flexible, as it will have to accommodate both the lateral movement of the roller along the cam as the current design does, as well as the movement of the roller due to the rotation of the follower. Because of this additional movement, the placement of the shifter becomes more critical. Guides similar to those in Figure 2 will need to be placed along the inactive portion of the cam in which the follower motion is constant along the cam's length. This corresponds to where the draft of the cam is zero, as illustrated in Figure 13. At this point, regardless of the particular profile the followers are on, the rollers and shifter ring will remain at a constant distance from the center and the shifter guide size can be minimized.



Figure 13. Points on the cam of zero draft, or where the shifter guides should be located.

# **5 CONCLUSIONS**

In this paper, several mechanisms and design approaches used to improve the torque and speed capacity of the unique Cam-based IVT have been presented. The number of active followers was doubled, allowing a greater distribution of the input reaction force, by the use of a compact, lightweight, and capable differential mechanism based on a cord and pulley system. The contact stress between the followers and the cam surface was reduced by 41% due to this mechanism alone. A kinematic model governing the motion of this differential was presented with a few experimental results from the prototype which showed a smooth output, a wide gear range, and shifting under load were all possible. Plans for future improvements to the design, including an external cam mechanism, were presented which is expected to reduce the contact stress another 21% through an improved contact geometry and component resizing. The torque output of the transmission is also expected to improve through a reduction in the return spring size made possible by the inverted, external cam.

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