Proceedings of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2009
August 30 - September 2, 2009, San Diego, California, USA

DETC2009-86912

ANALYSIS, SYNTHESIS, AND EXPERIMENTS OF STANDING UP METHODS FOR A TRIPEDAL ROBOT

Ivette M. Morazzani
RoMeLa: Robotics & Mechanisms Laboratory
Mechanical Engineering Department
Virginia Tech
Blacksburg, VA 24061
Email: imorazza@vt.edu

Dennis W. Hong
RoMeLa: Robotics & Mechanisms Laboratory
Mechanical Engineering Department
Virginia Tech
Blacksburg, VA 24061
Email: dhong@vt.edu

ABSTRACT

This paper presents the work addressing the issue of standing up after falling down for a novel three-legged mobile robot STriDER (Self-excited Tripedal Dynamic Experimental Robot). The robot is inherently stable when all three feet are on the ground due to its tripod stance, but it can still fall down if it trips while taking a step or if unexpected external forces act on it. The unique structure of STriDER makes the simple task of standing up challenging for a number of reasons; the high height of the robot and long limbs require high torque at the actuators due to its large moment arms; the joint configuration and length of the limbs limit the workspace where the feet can be placed on the ground for support; the compact design of the joints allows limited joint actuation motor output torque; three limbs do not allow extra support and stability in the process of standing up. This paper examines four standing up strategies unique to STriDER: three feet, two feet and one foot pushup, and spiral pushup. For all of these standing up strategies, the robot places its feet or foot at desired positions and then pushes the feet against the ground thus, lifting the body upwards. The four pushup methods for standing up were analyzed and evaluated considering the constraints such as, static stability, friction at the feet, kinematic configuration and joint motor torque limits, thus determining the suggested design and operation parameters. The motor torque trends as the robot stands up using pushup methods were investigated and the results from the analysis were validated through experiments.

INTRODUCTION AND MOTIVATION

STriDER can often fall down if it trips while walking or if external forces act on it. Thus, it is important to investigate a variety of standing up strategies specific to STriDER in order for the robot to stand up and complete its tasks. This paper focuses on four types of feet puhshup methods: three feet, two feet and one foot pushup, and spiral pushup. Generally, in all four methods, the robot first places its feet or foot at desired positions, then lifts its body by pushing the feet against the ground [1, 2]. The feet contact points are assumed to be stationary through the whole process.

The unique structure and operation of STriDER makes the simple task of standing up challenging for a number of reasons; the tall height and long limbs of the robot require high torque from the actuators due to large moment arms; the joint configuration and length of the limbs limit the workspace where the feet can be placed on the ground for support; the compact design of the joints allows for limited actuator torque; and the number of limbs (three) does not allow extra support and stability in the process of standing up. A detailed analysis of the feet pushup methods is presented in this paper considering constraints such as, static stability, friction at the feet, kinematic configuration, link length ratios, and actuator torque limits. The objective of this analysis is to determine optimal design and operation parameters that will minimize actuator torques as the robot stands up. By minimizing actuator torque less power is consumed and the robot can stand up more efficiently. In addition, due to the size and weight limitations of the robot, it is difficult to find the most powerful and ideal motors for STriDER. Thus, finding the parameters for minimum torque is important. Also it was assumed that dynamics will not have a large effect due to slow motions thus, all the analysis was based on quasi-static equilibrium assumptions [3]. Experiments are also presented in this paper to validate the analysis.

STriDER: Self-excited Tripedal Dynamic Experimental Robot

The novel tripedal gait (patent pending) of STriDER was discussed in detail in [4, 5]. During a step, two legs act as stance legs while the other acts as a swing leg. STriDER begins with a stable tripod stance then, the hip links are oriented to push the center of gravity forward by aligning the stance legs' pelvis links. As the body of the robot falls forward, the swing leg naturally swings in between the two stance legs and catches the fall. As the robot takes a step, the body needs to rotate 180 degrees to prevent the legs from tangling up. Once all three legs are in contact with the ground, the robot regains its stability and the posture of the robot is reset in preparation for the next step.

Also, a full three-dimensional kinematic model was developed to aid in the inverse and forward displacement analysis in the robot's triple stance phase. This model will help examine standing up strategies and is beneficial for visualizing the motion of STriDER's links and joints.

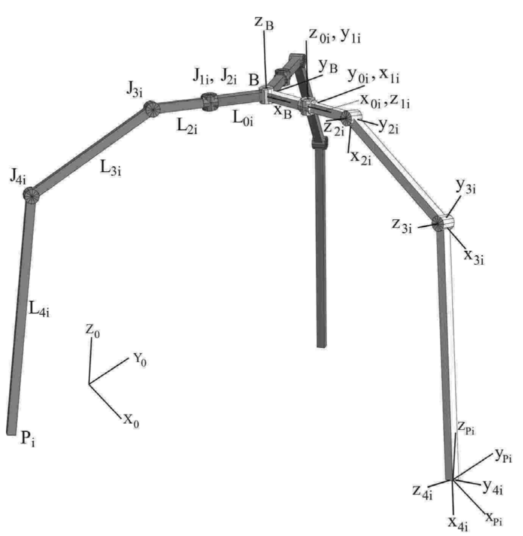


Figure 1. Coordinate frames and joint definitions for STriDER [6].

Enlightments from the Research on Human Motions

A variety of research has been conducted for human sitting to standing motion, as presented in [3, 7–9]. In particular, Hutchison et al. completed a dynamic analysis of joint forces and torques while rising from a chair. They concluded that quasi-static models (assume the body segments are in static equilibrium at any instant) are valid for chair-rise and dynamic effect is not the dominant factor in this motion. From Hutchison's findings [3], it was concluded that for the study of STriDER's standing up strategies, dynamics can be ignored and the analysis will be solely statically based, assuming the robot is not standing up at high speeds.

The human motion of standing up was also investigated by other researchers. Schenkman et. al. studied whole-body movements during rising to standing from sitting [9]. The goal of this study was to gain a brighter insight on the rising from a chair motion to facilitate identifying impairments of people who have trouble standing. In [7], an analysis of the sit-stand-sit movement cycle was done with normal subjects. Kerr et al. have obtained a basis of descriptive data for sit-stand-sit movement cycle from fifty normal subjects of various ages and both sexes [7]. A synthesis of standing up trajectories using dynamic optimization was presented in [8]. Kuẑeliĉki et al. utilized dynamic optimization as a tool to compute standing-up trajectories.

In addition to rising and standing findings, pushup exercise analysis was also considered in the research findings. First, Kai-Nan et. al investigated intersegmental loading patterns on the elbow joint during a push-up exercise [10]. They found that the hand position had a statistically significant effect on the axial force on the elbow. Then, in [11], Donkers et al. found that as the distance between the hand position increased the peak forces exerted on the elbow joint along a forearm axis decreased. Thus, it is easier to do a pushup when the hands are father apart on the floor. Therefore, the effects of different foot positions on the actuator torques as STriDER stands using the feet pushup must be investigated.

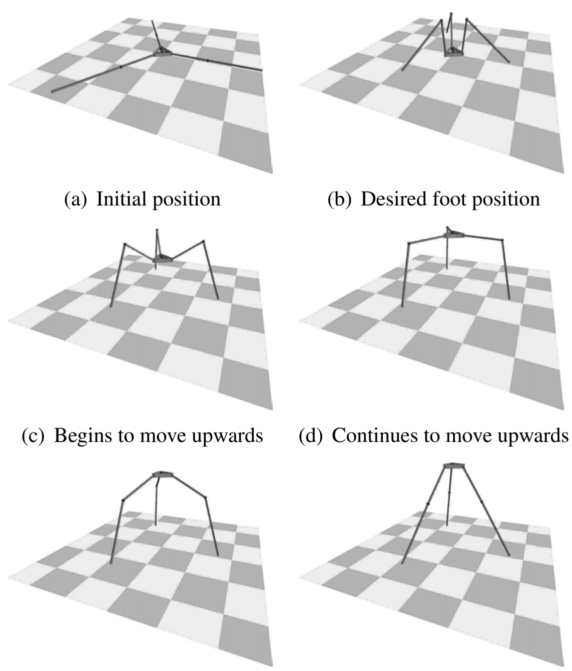
As noted, the results of the research findings influenced the steps for the investigation of STriDER's standing up strategy. In particular, it was concluded that dynamics would not have a large effect on the results thus, all analysis was statically based. Also, it was found that foot position is an important constraint when optimizing actuator torques.

STANDING UP METHODS

In the following sections, four types of standing up methods are examined one by one. All these four methods begin with the robot flat on the ground with all three legs extended outwards, then use the robot's feet to push up againt the ground. During the lifting of the body, all the contact points are assumed to be stationary.

THREE FEET PUSHUP

Beginning with the robot flat on the ground with all three legs extended outwards, as shown in Fig. 2(a), the three feet pushup method moves the three legs inwards towards the body to the final desired positions of the feet, thus forming an equilateral triangle. A distance d for one leg, shown in Fig. 3, is defined as the distance between the projected center of the body to the ground and the foot contact point. For this case, the distance d for all three legs is equal since the three contact points form an equilateral triangle. The value of d will play an important role in the motor torque calculations of the required motor torques at the joints. Once the feet reach their desired foot positions, the body begins to move upwards by pushing against the ground until it reaches its desired height (Fig. 2). The maximum body height is achieved when the thigh and shank links are aligned, as shown in Fig. 2(f).



(a) Initial position (b) Desired foot position

(c) Begins to move upwards (d) Continues to move upwards

(e) Continues to move upwards (f) Maximum height reached

Figure 2. The motion of the three feet pushup.

Mechanics of the Three Feet Pushup

Ihe symmetrical approach of three feet pushup allows for simpler analysis and guarantees static stability since the center of gravity is always located in the center of the body. The configuration for all three legs in the three feet pushup standing up method is the same thus, a detailed analysis for only one leg is presented here as the other two legs will follow the same procedure. A kinematic and quasi-static torque analysis is presented for the portion when the body begins to move upward and reaches its maximum height (Fig. 2(b) to Fig. 2(f)). The effect of d values, link length ratios ($\alpha = \frac{r_3}{r_4}$) and allowable tangential friction forces between the feet and the ground on the motor torques will be investigated in the analysis.

Kinematic Analysis To find the joint angles of the leg as the robot stands up, the body and links can be modeled as a slider-rocker mechanism, where the body is the slider link moving vertically, the thigh is the coupler link, and the shank is the rocker, as shown in Fig. 3. For a no slip condition, the foot contact point can be modeled as a revolute joint, between the shank link and the ground.

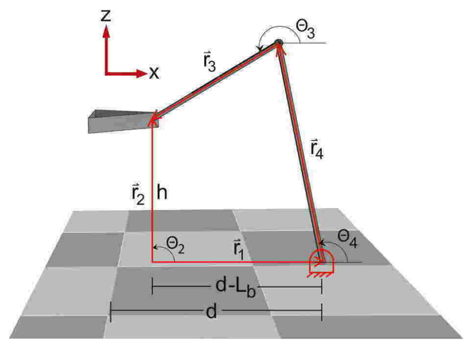


Figure 3. Body and one leg modeled as a slider-rocker mechanism.

As the body moves upward in the positive z direction, the joint angles, θ_3 and θ_4 are calculated given the body height, h, and using the vector loop equation, shown in Equation 1. The angle of vector $\vec{r}_1$, θ_1, equals zero and θ_2 equals 90 degrees, when the body is moving straight up perpendicular to the ground. The value of d is predefined, L_b is the constant body link length ($L_b = L_0 + L_1 + L_2$) (Fig. 1) and h is the input variable. With theses values, Equation 1 is simplified, thus θ_3 and θ_4 can be calculated using Equation 2. Also, the maximum height of the body, or the height when the thigh and shank link are aligned, is calculated with Equation 3.

$$\begin{aligned} x &: r_2cos\theta_2 - r_3cos\theta_3 - r_4cos\theta_4 - r_1cos\theta_1 = 0 \\ z &: r_2sin\theta_2 - r_3sin\theta_3 - r_4sin\theta_4 - r_1sin\theta_1 = 0 \end{aligned} \tag{1}$$

$$\begin{aligned} x &: -r_3cos\theta_3 - r_4cos\theta_4 - (d - L_b) = 0 \\ z &: h - r_3sin\theta_3 - r_4sin\theta_4 = 0 \end{aligned} \tag{2}$$

$$h_{max} = \sqrt{(r_3+r_4)^2-(d-L_b)^2} = \sqrt{(r_{tot})^2-(d-L_b)^2} \quad (3)$$

Static Force Analysis A free body diagram for the links of one leg is shown in Fig. 4. A friction force was added at the feet to account for different tangential forces, F_T. For example, if STriDER was attempting to stand up on ice, where there is very limited friction (small allowable tangential forces), the motor torque requirements at the joints would be different than those when standing up on a rough surface, where relatively larger tangential forces can exist. In fact, the tangential forces at the foot contact points can be adjusted to minimize the joint torque requirements.

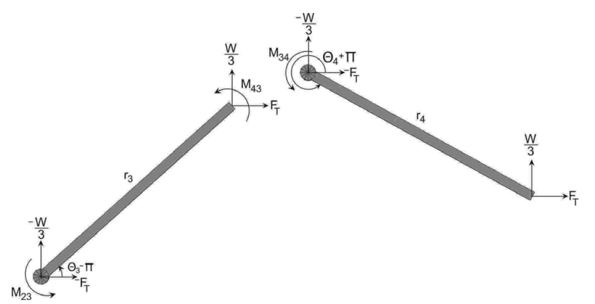


Figure 4. Free body diagram for one leg of the three feet pushup.

The moment at the flexure joint, M_{23}, and at the knee joint, M_{34}, are calculated from Equations 4 and 5, respectively. The legs were assumed to be weightless for the torque analysis since they would be negligible compared to the weight of the body. Also, $\theta_4+\pi$ and $\theta_3-\pi$ are the angles from the positive x-axis to the shank and thigh links, respectively. By combining the kinematic analysis using the slider-rocker mechanism and force analysis using the FBD equations, the torque at the flexure and knee joints can be calculated.

$$M_{34} = r_4 sin(\theta_4+\pi) F_T - r_4 cos(\theta_4+\pi)\frac{W}{3} \quad (4)$$

$$\begin{aligned} M_{23} &= r_3 sin(\theta_3-\pi) F_T - r_3 cos(\theta_3-\pi)\tfrac{W}{3} + M_{34} \\ &= -hF_T-(d-L_b)\tfrac{W}{3} \end{aligned} \quad (5)$$

Actuator Torque for the Three Feet Pushup

This section will investigate the effects of d, link length ratio ($\alpha = \frac{r_3}{r_4}$), and F_T on the actuator torques. In this case, a total link length (r_{tot}= r_3 + r_4) of 1.2 m, body link length, L_b, (=$L_0+L_1+L_2$, shown in Fig. 1) equal to 0.18 m are assumed. A maximum friction coefficient, μ, of 0.3 was chosen for this analysis and the total weight of the robot was set to 28.42 N. A tangential force, F_T, can be chosen for any value less than the normal force times the friction coefficient. Since only one leg is being analyzed the maximum tangential force is one third of the weight of the body times μ. The maximum magnitude of F_T is 2.84 N and can act in both the positive and negative direction, $-2.84N \leq F_T \leq 2.84N$. The minimum magnitude of the F_T is equal to zero. The positive and negative values of F_T account for an outwards and inwards reactive force at the feet. Based on Equations 4 and 5, the analysis first starts with the effects of d and α on the actuator torques when F_T equals zero. Once the effects of d and α on the actuator torques are investigated, the effects of F_T will be studied.

Effects of d on Actuator Torques The parameters chosen for the analysis were as follows: $r_3 = 0.45$ m, $r_4 = 0.75$ m, (or $\alpha = 0.6$) and d ranged from $(r_4-r_3)+L_b$ and $(r_3+r_4+L_b)$. As the value of d increases the maximum height of the body (h_{max}) decreases since the stance of the robot is larger. The flexure joint torque, decreases as d decreases. Also, when F_T is equal to zero the flexure joint torque is constant for each value of d ($M_{23} = \frac{-(d-L_b)W}{3}$). However, the knee joint torque significantly changes as d changes. For this example, the maximum knee joint torque occurs when d equals the minimum allowable value (0.48 m) and h=0. The maximum knee joint torque does not always occur when h equals zero as will be later discussed.

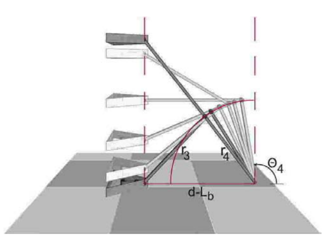
Effects of Link Length Ratio on Actuator Torque The effects of the link length ratio, α, were studied given a d value of 0.65 m and a total link length, r_{tot}, of 1.2 m and F_T equal to zero. The minimum and maximum allowable values of α given r_{tot}, d and L_b are calculated from Equations 6 and 7. Note that if α_{min} equals zero then d equals r_{tot} + L_b where r_{tot} equals r_4.

$$\alpha_{min} = \frac{-d+L_b+r_{tot}}{d-L_b+r_{tot}} \quad (6)$$

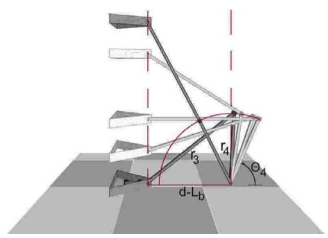
$$\alpha_{max} = -\frac{-d+L_b-r_{tot}}{-d+L_b+r_{tot}} \quad (7)$$

The flexure joint torque is not affected by the various link length ratios when a value for d is specified and F_T equals zero. From Equation 5, the flexure joint torque can be defined in terms of F_T, h, $d-L_b$, and W thus, for a given d value and total link length value, r_{tot}, the different link length ratios will not affect the flexure joint torque. The knee joint torque, however, is affected by α. As α increases and the robot stands up the knee joint experiences a smaller range of torques. Note that the maximum knee joint torque occurs at different heights for each α. In this case, the maximum knee joint torque occurs when α equals α_{min} and h equals 0.

Effects of the Tangential Force on Actuator Torques As mentioned above, a range of $-2.84N \leq F_T \leq 2.84N$ was considered given a maximum magnitude of 2.84 N ($\frac{\mu W}{3}$) acting in positive and negative directions at the feet. For this analysis, the three feet pushup can be divided in two other cases: *Case* 1 and *Case* 2 (separate from Case A, B and C, previously discussed), as shown in Fig. 5. *Case* 1 is defined as the case when $\frac{\pi}{2} < \theta_4 < \pi$ (Fig. 5(a)) during the entire standing motion and $r_3 < d - L_b$. *Case* 2 is defined as the case when θ_4 will equal $\frac{\pi}{2}$ at least once as the robot stands and $r_3 \geq d - L_b$ (Fig. 5(b)). The chosen parameters for *Case* 1 were r_3 = 0.45 m, r_4=0.75 m, and d = 0.65 m. For this case $d - L_b$ = 0.47 m. On the other hand, the chosen parameters for *Case* 2 were r_3 = 0.45 m, r_4=0.75 m, and d = 0.55 m, where $d - L_b$ = 0.37 m. In order to adequately compare the two cases the same link length ratio ($\alpha = 0.6$) was selected.



(a) *Case* 1: $r_3 < d - L_b$



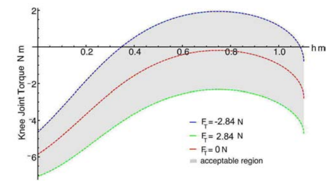
(b) *Case* 2: $r_3 \geq d - L_b$

Figure 5. Case study of the three feet pushup standing up method.

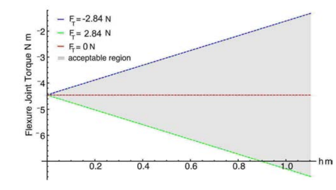
After comparing the actuator joint torque results given an allowable friction force range, defined by a friction coefficient and normal force, it was concluded that a maximum tangential force acting inwards towards the body will result in the lowest actuator torques as the body lifts using the three feet pushup.

Experiments

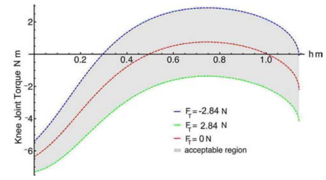
Experiments were conducted to validate the analysis of the three feet pushup method. Torque readings were recorded for the flexure and knee joints at twenty different heights for seven trials. All torque values were recorded from static positions thus, the values were not recorded continuously as the robot stood up. The same link lengths and testing parameters were chosen for all of the trials. These parameters included; a thigh link length, r_3, equal to 0.495 m, a shank link length, r_4, equal to 0.56 m, and a d value of 0.67 m. However, the torque feedback fluctuates greatly due to various variables thus, the readings are not very accurate. Although the results of the experiments cannot be used to directly compare the analytical and experimental data, experiments were conducted to determined actuator troque trends.



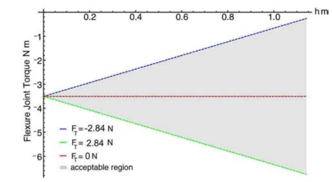
(a) *Case* 1: Knee joint torque range



(b) *Case* 1: Flexure joint torque range



(c) *Case* 2: Knee joint torque range



(d) *Case* 2: Flexure joint torque range

Figure 6. Joint torque region defined by a maximum and minimum F_T for the three feet pushup.

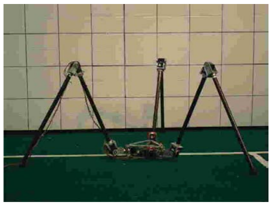
Fig. 7 shows various positions as STriDER stands using the three feet pushup method. As noted, the actuator torques were recorded from the motor feedbacks at twenty different heights for seven trials. The average of the seven trials was used to determine the actuator torque trends as the robot stands.

The results of the three feet pushup experiments are shown in Fig. 8. As shown in Fig. 8(a), the flexure joint torque followed a close trend for all seven trials. At first, the flexure joint experiences a large change in torque and then slowly decreases. This validates why it is difficult for the robot to stand from a height of zero.

The knee joint torque, shown in Fig. 8(b), however, does not follow such a close trend. It is important to note that although the motor torque readings were not as consistent as it would have been liked, they still can show an adequate trend. The maximum knee joint torque occurs near the end as the robot stands. However, after comparing the flexure joint and knee joint experimental results, the flexure joint experiences the largest torque. Also, the changes in torque show the effects of the tangential force on the actuator torques.

Spiral Pushup

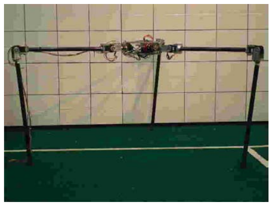
The spiral pushup method begins with the robot flat on the ground with all three legs extended outwards, as shown in Fig. 9(a). The feet are then positioned to a desired final foot position, as shown in Fig. 9(b). The final foot position is defined by a



(a) Desired foot position

(b) Begins to move upwards

(c) Continues to move upwards

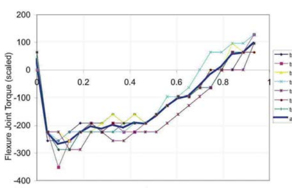


(d) Continues to move upwards

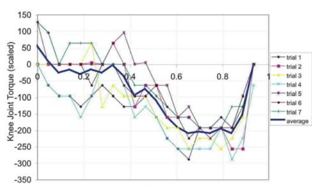
(e) Continues to move upwards

(f) Maximum height reached

Figure 7. Experiments of the three feet pushup.



(a) Flexure joint torque

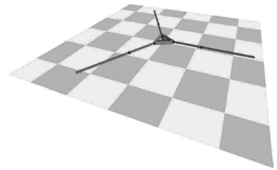


(b) Knee joint torque

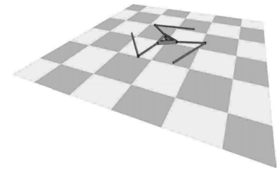
Figure 8. Three feet pushup joint torque experiment results (r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

distance d (distance between the projected center of the body on the ground and foot position) and desired maximum body rotation about the $+Z_0$ axis. Once the feet are located at the desired final feet position, the body is lifted upwards by actuating the rotator joints and the legs pushing against the ground (Figs. 9(c) to 9(e)). The robot continues to lift and rotate about the $+Z_0$ axis until it reaches a maximum height, as shown in Fig. 9(f).

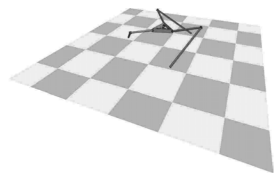
Note that the foot positions of the spiral pushup form an equilateral triangle. In fact, the configuration for all three legs in the spiral pushup standing up method is the same thus, a detailed analysis for only one leg is presented here as the other two legs will follow the same procedure. A kinematic and static force analysis is presented for the portion when the body begins to



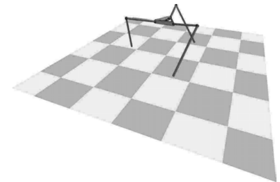
(a) Initial position



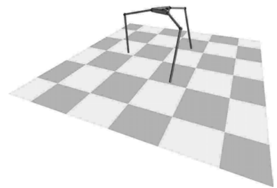
(b) Desired foot position



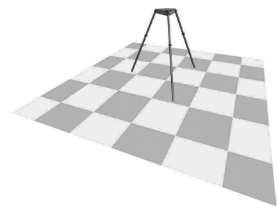
(c) Body begins to twist and lift



(d) Robot continues to stand up



(e) Legs begin to straighten



(f) Maximum height reached

Figure 9. The motion of the spiral pushup.

move upwards and twist and reaches its maximum height (Fig. 9(b) to Fig. 9(f)) for a range of d values, a range of allowable tangential friction forces between the feet and the ground, a range of maximum body rotations about the $+Z_0$ axis, and various link length ratios, α. The distance between the projected center of the body on the ground and final desired foot position is defined as d. As mentioned, a range of tangential contact force between the foot and the ground is defined by the friction coefficient and the normal contact force due to gravity. The body rotation about $+Z_0$ will range from zero (no rotation) to a desired maximum body rotation as the robot stands. In this example, the body rotates about the $+Z_0$ in a clockwise direction, thus the angles will be negative. The effects of these parameters on the actuator torques will be studied.

Kinematic Analysis for the Spiral Pushup

In order to obtain the rotator, flexure and knee joint angles as the robot stands using the spiral pushup method, a d value and maximum body rotation ($\theta_{Z_0 max}$ about $+Z_0$ axis are defined. Fig. 10 shows a top view of the robot when the feet are located at their desired final foot positions. As mentioned, the final foot positions are calculated from d and $\theta_{Z_0 max}$ using Equations 8 and 9. Recall that $\theta_{Z_0 max}$ is negative since the body rotates in a clockwise direction. Also, the maximum allowable height for a given d value is defined when the thigh and shank links are aligned.

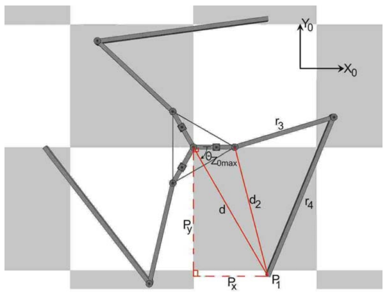
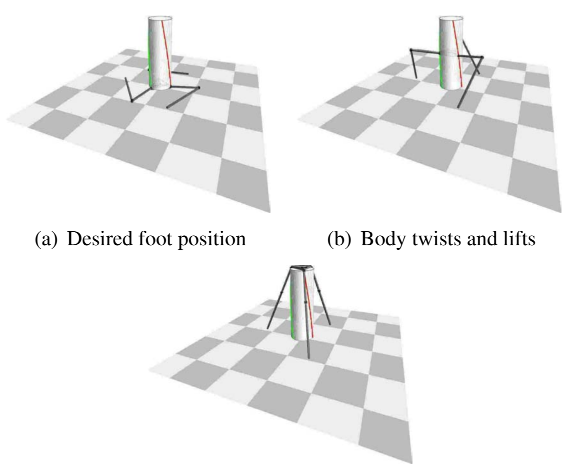


Figure 10. Top view of the initial position for spiral pushup kinematic and static force analysis.

$$P_x = d sin\left(\frac{\pi}{2} + \theta_{Z_0max}\right) \tag{8}$$

$$P_y = -d cos\left(\frac{\pi}{2} + \theta_{Z_0max}\right) \tag{9}$$

As the body height increases the current body rotation about the $+Z_0$ axis, θ_{Z_0}, can be calculated using Equation 10, where h is the current body height, θ_{Z_0max} is the maximum body rotation along the $+Z_0$ and is a constant (negative), and h_{max} is the maximum height when the thigh and shank links are aligned. In fact, the trajectory of the flexure joint forms a helix defined by the body height and $+Z_0$ rotation, as shown in Fig. 11.



(a) Desired foot position

(b) Body twists and lifts

(c) Maximum height

Figure 11. The flexure joint trajectory follows a helical shape.

$$\theta_{Z_0} = h\frac{\theta_{Z_0max}}{h_{max}} \tag{10}$$

Since the global body position and orientation and the global foot positions are known, (assuming the global coordinates are located in the initial center of the body location) the rotator, flexure and knee joint angles can be calculated using inverse kinematics.

Static Force Analysis for the Spiral Pushup

The rotator, flexure and knee joint torques are calculated using a static force analysis. A general torque expression, shown in Equation 11, will be used to find the individual joint torque equations. This equation shows that torque can be calculated by taking the cross product of a distance vector from the feet to the joint with a force vector at the feet and then taking the dot product of that result with the a unit vector at the desired joint. The distance vector components (x, y, z) from the foot of leg 1 to the rotator, flexure and knee joints as the robot stands are found using Equations 12, 13, and 14, where P_x is the X_0 distance from the project center of the body to the foot position, P_y is the Y_0 distance from the project center of the body on the ground to the foot position, L_0 is the distance between the center of the body to the rotator joint, θ_{Z_0} is the body rotation about the Z_0 axis, r_3 is the thigh link length, θ_{21} is the rotator joint angle for leg 1, and θ_{31} is the flexure joint angle for leg 1.

$$Torque = \left[\{dx_P^j, dy_P^j, dz_P^j\} \times \{F_x, F_y, F_z\}\right] \cdot \vec{n}_j \tag{11}$$

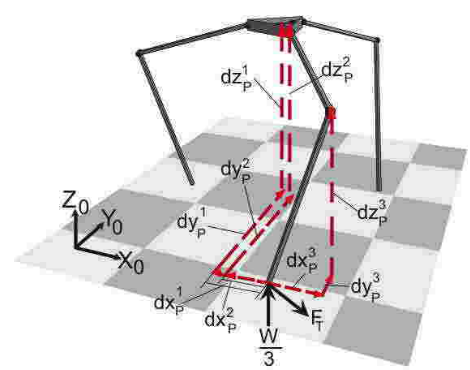


Figure 12. Foot to joint position distance labels.

$$\mathbf{d}_P^{11} = \begin{bmatrix} dx_P^{11} \\ dy_P^{11} \\ dz_P^{11} \end{bmatrix} = \begin{bmatrix} -P_x + L_0 cos(\theta_{Z_0}) \\ -P_y + L_0 sin(\theta_{Z_0}) \\ h \end{bmatrix} \tag{12}$$

$$\mathbf{d}_P^{21} = \begin{bmatrix} dx_P^{21} \\ dy_P^{21} \\ dz_P^{21} \end{bmatrix} = \begin{bmatrix} -P_x + L_b cos(\theta_{Z_0}) \\ -P_y + L_b sin(\theta_{Z_0}) \\ h \end{bmatrix} \tag{13}$$

$$\mathbf{d}_P^{31} = \begin{bmatrix} dx_P^{31} \\ dy_P^{31} \\ dz_P^{31} \end{bmatrix} = \begin{bmatrix} -P_x + cos(\theta_{Z_0})(L_b + r_3 sin(\theta_{31})) \\ -r_3 cos(\theta_{31}) sin(\theta_{Z_0}) sin(\theta_{21}) \\ \\ -P_y + sin(\theta_{Z_0})(L_b + r_3 sin(\theta_{31})) \\ +r_3 cos(\theta_{31}) cos(\theta_{Z_0}) sin(\theta_{21}) \\ \\ h - r_3 cos(\theta_{31}) cos(\theta_{21}) \end{bmatrix} \tag{14}$$

Also, the tangential force components (x and y) at the feet can be found from Equation 15, where W is the total weight of the robot and μ is the friction coefficient. There is also a normal force at each foot equal to $\frac{W}{3}$. Note that the determined tangential force at the foot is always parallel to the body link for each leg. Thus, the components of the tangential force are defined by the angle of rotation, θ_{Z_0}, of the body. Although other tangential forces exist at the foot, they are assumed to cancel each other out and will not be considered in this analysis. Fig. 12 shows the components of the direction vector for each joint and the chosen direction of the tangential force at the foot.

$$F_{T_1} = \begin{bmatrix} F_{T_{x_1}} \\ F_{T_{y_1}} \end{bmatrix} = \begin{bmatrix} \frac{\mu W cos(-\theta_{Z_0})}{3} \\ \frac{-\mu W sin(-\theta_{Z_0})}{3} \end{bmatrix} \tag{15}$$

From the general torque equation and the unit vector at the rotator joint found using forward kinematics, the rotator joint torque can be calculating using Equation 16,

$$M_{12} = \frac{1}{3} W \left[dy_P^{11} cos(\theta_{Z_0}) - dx_P^{11} sin(\theta_{Z_0}) \right] \tag{16}$$

where dx_P^{11}, dy_P^{21}, dz_P^{11} are the x, y and z vector components from the foot position of leg 1 to the rotator joint, θ_{Z_0} is the body rotation about the $+Z_0$ axis in the clockwise direction, and W is the total weight of the body. Note that because the chosen tangential force and the rotator joint have the same unit vector, F_T will does not affect the torque at the rotator joint.

Next, the flexure joint torque can be calculated using Equation 17,

$$M_{23} = \tfrac{1}{3} cos(\theta_{21}) \left[-3 dz_P^{21} F_T + dx_P^{21} W cos(\theta_{Z_0}) + dy_P^{21} W sin(\theta_{Z_0}) \right] + F_T sin(\theta_{21}) \left[dy_P^{21} cos(\theta_{Z_0}) - dx_P^{21} sin(\theta_{Z_0}) \right] \tag{17}$$

where dx_P^{21}, dy_P^{21}, dz_P^{21} are the x, y and z vector components from the foot position of leg 1 to the flexure joint, θ_{Z_0} is the body rotation about the $+Z_0$ axis in the clockwise direction, θ_{21} is the rotator joint angle for leg 1, and W is the total weight of the body.

Lastly, the knee joint torque can be calculated using Equation 18,

$$M_{34} = \tfrac{1}{3} cos(\theta_{21}) \left[-3 dz_P^{31} F_T + dx_P^{31} W cos(\theta_{Z_0}) + dy_P^{31} W sin(\theta_{Z_0}) \right] + F_T sin(\theta_{21}) \left[dy_P^{31} cos(\theta_{Z_0}) - dx_P^{31} sin(\theta_{Z_0}) \right] \tag{18}$$

where dx_P^{31}, dy_P^{31}, dz_P^{31} are the x, y and z vector components from the foot position for leg 1 to the knee joint, θ_{Z_0} is the body rotation about the $+Z_0$ axis in the clockwise direction, θ_{21} is the rotator joint angle for leg 1, and W is the total weight of the body.

Actuator Torque for Spiral Pushup

The effects of F_T, d, and link ratios ($\alpha = \frac{r_3}{r_4}$) on the actuator torques will be studied in this section. A given total link length (r_{tot}=r_3 + r_4) of 1.2 m was chosen for the analysis and d_2 (shown in Fig. 10) can be calculated from Equation 19 using the law of cosines.

$$d_2 = \sqrt{\left((L_b^2 + d^2) - \left(2L_b^2 d cos(-Z_{0max})\right)\right)} \tag{19}$$

Effects of d on actuator torques The effects of different d values on the actuator torques as the robot stands using the spiral pushup were studied. The following parameters were used; r_3 = 0.45 m, r_4=0.75 m, and $\theta_{Z_0 max}$= $-\frac{\pi}{3}$. Recall that, L_0, L_1, and L_2 will equal 0.1m, 0 m, and 0.08 m, for all of the standing up strategy analysis. The minimum and maximum allowable d values were found using Equations 20 and 21.

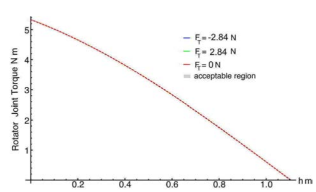
$$d_{min} = (L_0 + L_1 + L_2) cos(-Z_{0max}) + \sqrt{-L_0{}^2 - 2L_0 L_2 - L_2{}^2 + r_3{}^2 - 2r_3 r_4 + r_4{}^2 + (L_0 + L_2)^2 cos(-\theta_{Z_0 max})^2} \tag{20}$$

$$d_{max} = (L_0 + L_1 + L_2)\,cos\,(-Z_{0max}) + \sqrt{-L_0{}^2 - 2L_0L_2 - L_2{}^2 + r_3{}^2 + 2r_3r_4 + r_4{}^2 + (L_0 + L_2)^2 cos\,(-\theta_{Z_0max})^2} \quad (21)$$

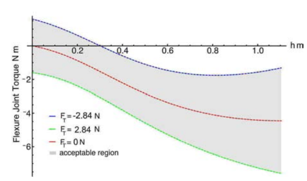
As the value of d increases the allowable maximum height of the body decreases since the stance of the robot is larger. The rotator joint torque increases as d increases. This trend is the same for the flexure and knee joints. Thus, it may be concluded that the actuator torques increase as the d increases for the spiral pushup. However, although a smaller d value will yield less torque at the actuators the robot will become more unstable since the support triangle formed by the foot contact points is smaller.

Effects of link length ratio on actuator torques The rotator and flexure joint torques do not change as α changes. However, the knee joint torque is affected by α. The optimal link length ratio yields the minimum maximum torque as the robot stands. From the given parameters the α that will yield the minimum maximum knee joint torque is equal to 1.01.

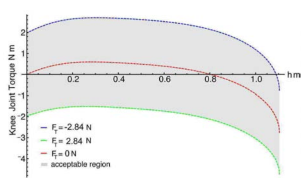
Effects of the tangential force on actuator torques The effects of a friction force at the feet on the actuator torques was studied for the spiral method as the robot stands. As mentioned ahead, a range of $-2.84N \leq F_T \leq 2.84N$ was considered given a maximum magnitude of 2.84 N ($\frac{\mu W}{3}$) acting in positive and negative directions at the feet. The chosen parameters were r_3 = 0.45 m, r_4=0.75 m, d = 0.65 m, and θ_{Z_0max}= $-\frac{\pi}{3}$. Fig. 13 shows the rotator, flexure and knee joint torque results for the defined F_T range.



(a) Rotator joint torque



(b) Flexure joint torque



(c) Knee joint torque

Figure 13. Actuator torque results for various F_T values for the spiral pushup.

As shown in Fig. 13(a), F_T does not affect the rotator joint torque since the unit vector at the rotator joint and the tangential force vector are in the same direction. The magnitude of the flexure joint torque is less for a tangential force acting inwards towards the center of the body than a tangential force acting outwards, as shown in Fig. 13(b). However, for the knee joint torque, the minimum torque will occur for a tangential force between 0 and -2.84 N.

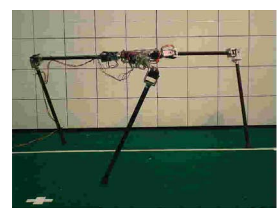
Experiments

Several static positions of the spiral pushup are shown in Fig. 14. As previously discussed, all three legs act the same for this method. First the feet are placed in their desired final positions, forming an equilateral triangle. Then, the body is lifted upwards by pushing the feet against the ground and rotating about the $+Z_0$ axis.


(a) Desired foot position


(b) Begins to move upwards


(c) Continues to move upwards


(d) Continues to move upwards

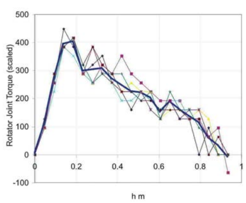

(e) Continues to move upwards


(f) Maximum height reached

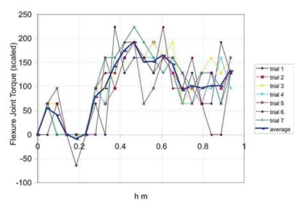
Figure 14. Experiments of the spiral pushup.

The results of the spiral pushup experiments are shown in Fig. 15. In addition to parameters listed above, a maximum body rotation of $\frac{-\pi}{6}$ was chosen for the spiral pushup experiments. The rotator joint results, presented in Fig. 15(a), show that the rotator joint experiences a large change in torque as the robot initially stands up but then decreases for the remainder of the body lifting. Next, the flexure joint torque, shown in Fig.

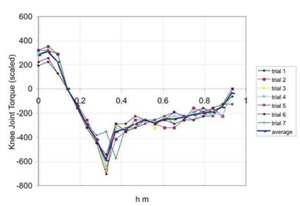
15(b), shows that the flexure joint torque varied some for the seven different trials. The trends shows that at first the flexure joint torque slightly increases, slightly decreases until it reaches close to zero, and then increases to a maximum torque value, and finally decreases. Finally, the knee joint torques were recorded for the knee joint, as the robot stands using the spiral pushup method, as shown in Fig. 15(c). These results show that the seven trials followed a very similar trend. The torque begins to decrease until it reaches zero, then increases in magnitude until the maximum torque is reached. Once the maximum torque is reached, the torque decreases in magnitude and the maximum height is achieved. From the experiments, it can be concluded that the maximum joint torque occurs at the knee joint torque.



(a) Rotator joint torque



(b) Flexure joint torque

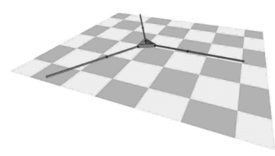


(c) Knee joint torque

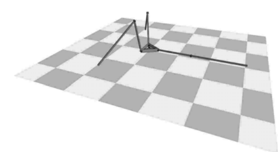
Figure 15. Spiral pushup joint torque experiment results (r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

TWO FEET PUSHUP

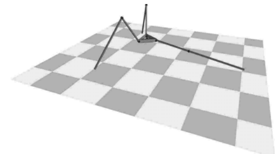
The two feet pushup begins with the robot flat on the ground with all three legs extended outwards (Fig. 16(a)) then, two of its legs move inwards toward the body to their final desired position, defined by d, leaving one leg extended (Fig. 16(b)). d is the distance from the center of the body to the desired final foot positions for the two bending legs. Once the two legs reach their desired foot positions, the body is pushed upwards by the two legs pushing against the ground until it reaches its maximum height (Fig. 16(c) to 16(e)). Note that the flexure joint of the straight leg follows an arc defined by the thigh and shank links as the robot stands. The maximum body height is achieved when the thigh and shank links of the bending legs are aligned. Although this method is statically stable always since all three feet are touching the ground and the projected center of gravity lies inside the support triangle, it requires high torques at the actuators due to the large moment arms. Again, the inverse kinematics will be used to find the joint angles for the bending legs and the straight leg. Similar to three feet spiral pushup, Equation 11 is used to determine the individual joint torque. Parameters such as d, link length ratio ($\alpha = \frac{r_3}{r_4}$), and F_T all have effects on the actuator torques. Detailed discussion can be found in [1, 2].



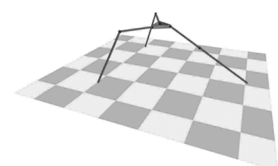
(a) Initial position



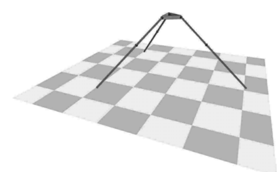
(b) Desired foot position



(c) Begins to move upwards



(d) Continues to move upwards



(e) Maximum height reached

Figure 16. The motion of the two feet pushup.

Two Feet Pushup Experiments

Various static positions of STriDER using the two feet pushup are shown in Fig. 17. Note that for this method two legs push the body upwards while the middle leg remains straight. Also, the foot positions do not change as the robot stands and they do not form and equilateral triangle. Lastly, the robot is always statically stable as is stands up using this strategy.

The results of the two feet pushup experiments are shown in Fig. 18 and 19. Similar, to the analytical analysis, the experiments were divided in two parts: leg 1 and leg 2. Leg 1 is the leg that remains straight as the robot stands, and leg 2 bends and pushes the body upwards as discussed before.

The flexure joint torque of the straight leg (leg 1), shown in Fig. 18(a), experiences relatively high torques as the robot stands. The flexure joint torque rapidly increases to a maximum value and then decreases until the robot reaches its maximum height. The knee joint of leg 1, shown in Fig. 18(b), first rapidly increases and then slowly decreases. Note that the flexure joint

(a) Desired foot position

(b) Begins to move upwards

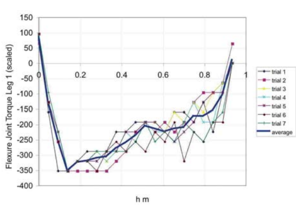
(c) Continues to move upwards

(d) Continues to move upwards

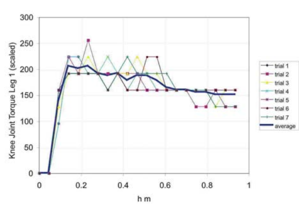
(e) Continues to move upwards

(f) Maximum height reached

Figure 17. Experiments of the two feet pushup.



(a) Flexure joint torque



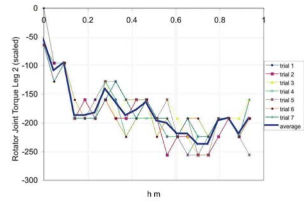
(b) Knee joint torque

Figure 18. Two feet pushup joint torque experiment results leg 1(r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

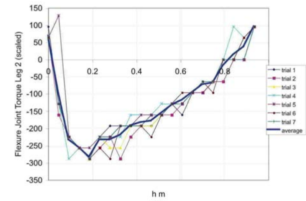
torque has a larger magnitude at the majority of the heights than the knee joint for leg 1.

The rotator joint torque of the bending leg (leg 2) is shown in Fig. 19(a). It was found that as the robot stands, the rotator joint experiences much higher torques. Next, the flexure joint torque was recorded and the results are shown in Fig. 19(b). As shown, the flexure joint torque for leg 2 rapidly increases, then slowly decreases, reaches zero and then slowly increases in magnitude again. The knee joint torque of leg 2 is shown in Fig. 19(c). In this case, the torque decreases, reaches zero, increases and then final decreases until it reaches a maximum height.

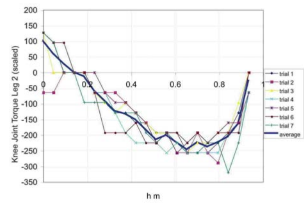
After analyzing the straight and bending leg experimental results, it was found that the maximum torque occurs at the flexure joint of the straight leg, as expected from the analysis.



(a) Rotator joint torque



(b) Flexure joint torque



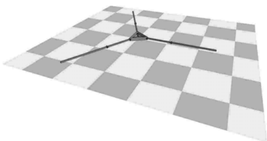
(c) Knee joint torque

Figure 19. Two feet pushup joint torque experiment results leg 2(r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

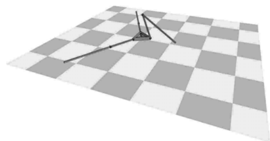
One Foot Pushup

The one foot pushup begins with all three legs straight and flat on the ground, as shown in Fig. 20(a). Then, one leg moves inwards toward the body to a final desired final foot position as the other two legs remain straight on the ground (Fig. 20(b)). Next, the body is pushed upwards by the bending leg until it reaches a maximum height (Fig. 20(c) to 20(f)). Note that the feet do not move once the bending leg's foot is in the desired position. Thus, as the body lifts upwards the initially straight legs must bend their knees so the feet are kept in their initial positions. As noted before, the maximum height is reached when the thigh and shank links of the all three legs are aligned. This method is also always statically stable since all three feet are touching the ground and the projected center of gravity lies inside the support triangle. The analysis of one foot pushup is very similar to two feet pushup. The inverse kinematics will be used to find the joint angles for the bending legs and the straight leg. Equation 11 is used to determine the individual joint torque. Again, the detailed discussion on effects of the parameters such as d, link length ratio ($\alpha = \frac{r_3}{r_4}$), and F_T on the actuator torques can be found in [1,2].

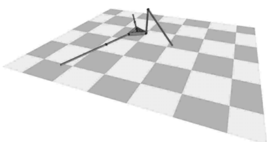
The configuration of the two straight legs (Fig. 20(b)) in the one foot pushup is the same, while the third leg (bending leg) is positioned at a desired foot position defined by d. Thus, the analysis of the one foot pushup is divided in two parts: analysis of the straight legs and analysis of the bending leg. A kinematic and torque analysis is presented for the portion when the body begins to move upwards and reaches its maximum height (Fig. 20(b) to 20(f)). A range of d values, various link length ratios ($\alpha = \frac{r_3}{r_4}$) and a range of allowable tangential friction forces be-



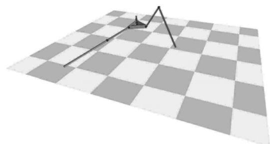
(a) Initial position



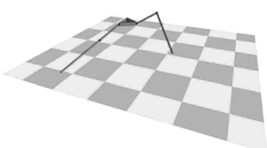
(b) Desired foot position



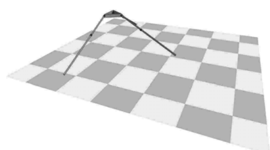
(c) Begins to move upwards



(d) Continues to move upwards



(e) Continues to move upwards



(f) Maximum height reached

Figure 20. The motion of the one foot pushup.

tween the feet and the ground will also be investigated in the analysis. As stated in the sections of three feet pushup and two feet pushup, the minimum allowable d is the difference between the thigh and shank link length plus the body link length. The maximum allowable d is the added length of the thigh, shank, and L_b. The range of tangential contact force between the foot and the ground is defined by the friction coefficient and the normal contact force due to gravity. Thus, the minimum tangential contact force is zero (no friction force) and the maximum tangential contact force is the normal force times the friction coefficient for a non-slip condition. As long as the tangential contact forces, at the three feet, satisfy these conditions and the force balance is satisfied, the tangential forces can be adjusted by force control of the actuators of the robot. The choice of the tangential force will effect the motor torque requirements at the joints.

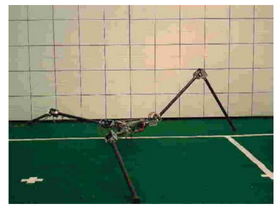
One Foot Pushup Experiments

Fig. 21 shows STriDER standing up using the one foot pushup method. As noted in the beginning of this section, for this strategy the middle leg's foot is moved to its desired final foot position defined by d while the other two legs do not bend. After some initial experiments is was determined that the actuator torques were to large for the motors to handles and they were braking. Thus, the knees of the straight leg were bent in order to complete the experiments and reduce the actuator torques as the robot lifted using a modified one foot pushup.

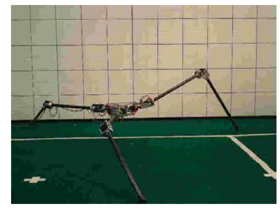
The results of the one foot pushup experiments are shown in Figs. 22 and 23. Similar to the two feet pushup experiments, the one foot pushup was also divided in two parts: bending and straight leg experiments. First, the flexure joint torques were

(a) Desired foot position

(b) Begins to move upwards



(c) Continues to move upwards



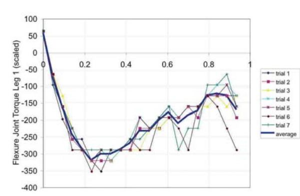
(d) Continues to move upwards

(e) Continues to move upwards

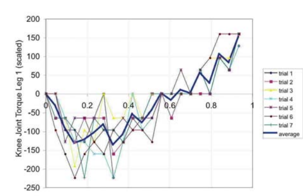
(f) Maximum height reached

Figure 21. Experiments of the one foot pushup.

recorded at twenty different heights for seven trials, as shown in Fig. 22(a). The flexure joint torque decreases until it reaches zero, and then increases until it reaches a maximum torque. After the maximum flexure joint torque for leg 1 is reached, the joint torque decreases in magnitude. Next, the knee joint torques were recorded for leg 1, as shown in Fig. 22(b). The knee torque increases in magnitude, then decreases, reaches zero and finally increases until the robot reaches its maximum height.



(a) Flexure joint torque

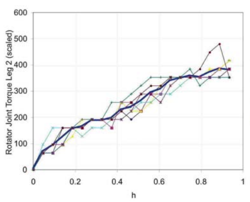


(b) Knee joint torque

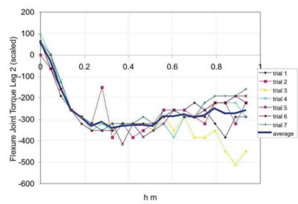
Figure 22. One foot pushup joint torque experiment results eg 1(r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

The experiment results of the rotator joint for leg 2 are shown in Fig. 23(a). The rotator joint torque increases until the maximum height is reached. Next, the flexure joint results are presented in Fig. 23(b). As shown, the flexure joint torque increases and then remains close to constant. Finally, the results of the knee joint torque experiments for leg 2 are shown in Fig. 23(c). In this case, the knee joint torque increases, reaches a maximum, decreases until it reaches zero and then increases until the maximum height it reached.

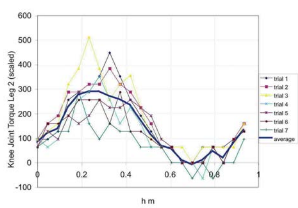
For the one foot pushup case, the maximum torque occurs at the rotator joint of leg 2.



(a) Rotator joint torque



(b) Flexure joint torque



(c) Knee joint torque

Figure 23. One foot pushup joint torque experiment results leg 2(r_3= 0.495 m, r_4=0.56 m, d = 0.67 m).

CONCLUSIONS

Four types of standing up methods are presented in this paper, which include three, two feet and one foot pushup and spiral pushup. The equations of the joint torques are developed under the quasi-static condition and the effects of the parameters such as d, α and friction force F_T are discussed. All four methods can be successfully implemented to the lastest prototype, as shown in the experimental results. Also, it was determined that of the four methods discussed in the paper the three feet pushup is both most efficient and the easiest to implement on a working prototype in terms of minimum torque requirements at the joints. The results derived will be used in the geometry design of future versions of STriDER.

REFERENCES

[1] Morazzani, I., and Hong, D., 2007. "Investigation of standing up strategies for a novel three-legged walking robot". *US-Korea Conference*, August.

[2] Morazzani, I., 2008. "Investigation of standing up strategies and considerations for gait planning for a novel three-legged mobile robot". Master's thesis, Virginia Polytechnic and State University, May.

[3] Hutchinson, E. B., Riley, P. O., and Krebs, D. E., 1994. "A dynamic analysis of the joint forces and torques during rising from chair". *IEEE Transactions on Rehabilitation Engineering,* **2**(2), June, pp. 49–55.

[4] Heaston, J., 2006. "Design of a novel tripedal locomotion robot and simulation of a dynamic gait for a single step". Masters thesis, Virginia Polytechnic and State University.

[5] Heaston, J., and Hong, D., 2007. "Design of a novel tripedal locomotion robot and simulation of a dynamic gait for a single step". *ASME Mechanisms and Robotics Conference*, September.

[6] Ren, P., Morazzani, I., and Hong, D., 2006. "Forward and inverse displacement analysis of a novel three-legged mobile robot based on the kinematics of in-parallel manipulators". *31st ASME Mechanisms and Robotics Conference*, September.

[7] Kerr, K. M., White, J. A., Barr, D. A., and Mollan, R. A. B., 1997. "Analysis of the sit-stand-sit movement cycle in normal subjects". *Clinical Biomechanics,* **12**(4), pp. 236–245.

[8] Kuzelicki, J., Zefran, M., Burger, H., and Bajd, T., 2003. "Synthesis of standing-up trajectories using dynamic optimization". *Science Direct*, November.

[9] Schenkman, M., Berger, R. A., Riley, P. O., Mann, R. W., and Hodge, W. A., 1990. "Whole-body movements during rising to standing from sitting". *Physical Therapy,* **70**(10), pp. 51–64.

[10] An, K.-N., Donkers, M. J., Chao, E. Y., and Morrey, B. F., 1992. "Intersegmental elbow joint load during pushup". *Biomedical Sciences Instrumentation*, pp. 69–74.

[11] Donkers, M. J., An, K.-N., Chao, E. Y., and Morrey, B. F., 1993. "Hand position affects elbow joint load during pushup exercise". *Journal of Biomechanics,* **26**(6), pp. 625–632.